# Physics, Measurements, and Numerical Modeling

## -- The Italian Connections

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# Outlines

I. Who was Lagrange and what Italian Connection?

II. Recent Italian Connection -- TRIM family of models

III. Using Italian Tools -- UnTRIM Wind-Driven Circulation in Upper Klamath Lake

# Joseph Louis Lagrange

## (Giuseppe Luigi Lagrangia)

## 1736-1813

**1766: Frederick the Great (Berlin) recruited him to take the position vacated by Euler, as the court mathematician**

**1787: Louis XVI invited him to Paris**

**Mechanique Analytique:**

**To unite and present from one point of view the different principles in mechanics**

Lagrangian point of view:

Reference frame is enclosing the mass.

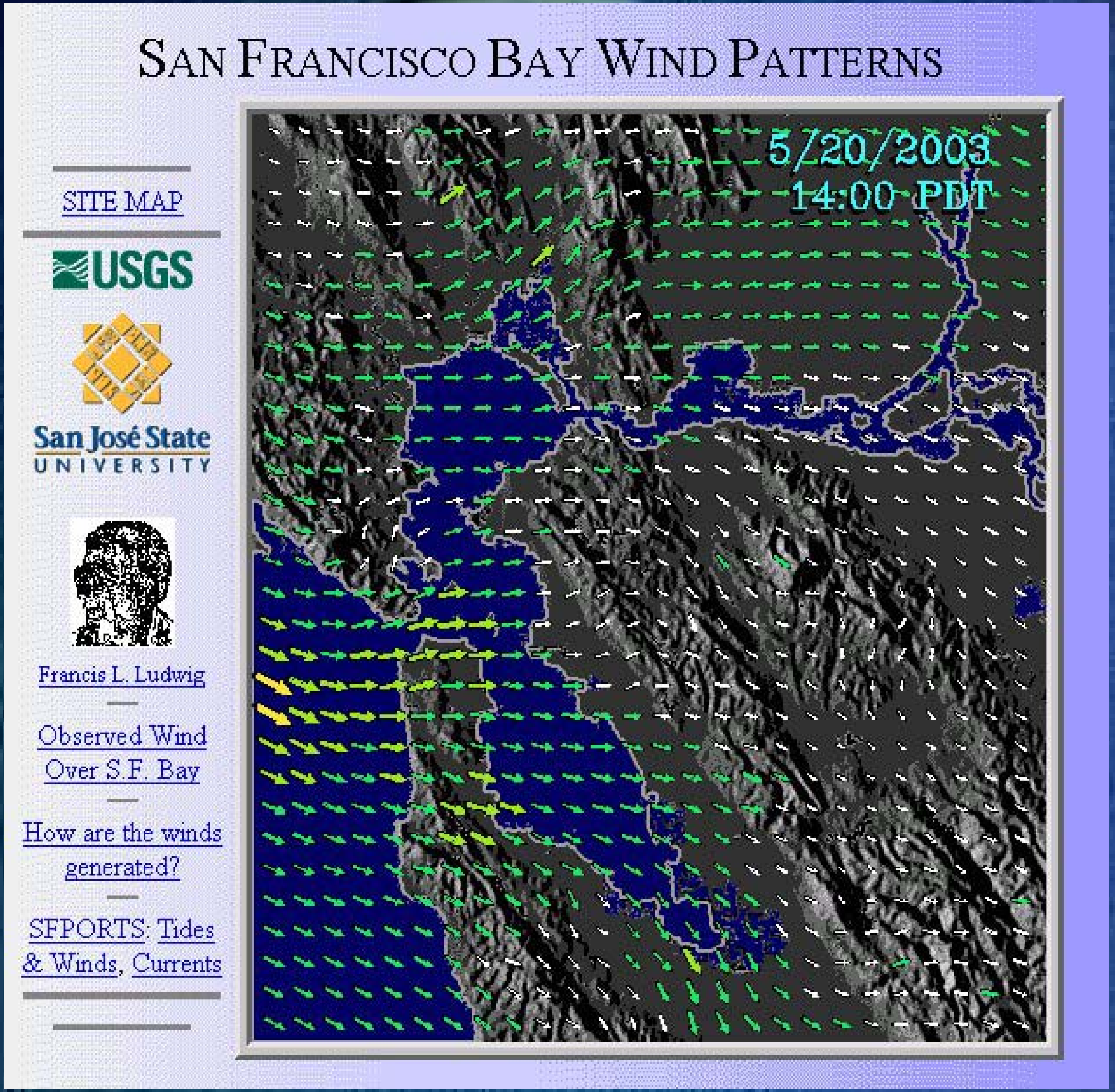
The coordinates are moving with the center of the mass.

Eulerian point of view:

Reference frame is fixed in space, the mass travels through the control volume.

The coordinates are fixed in space.

# Eulerian Representation




**Eulerian Variable:** $\theta = \theta(x, y, z, t)$

# Lagrangian Representation

## San Francisco Bay Wind Pattern Streaklines

SITE MAP

USGS

San José State University

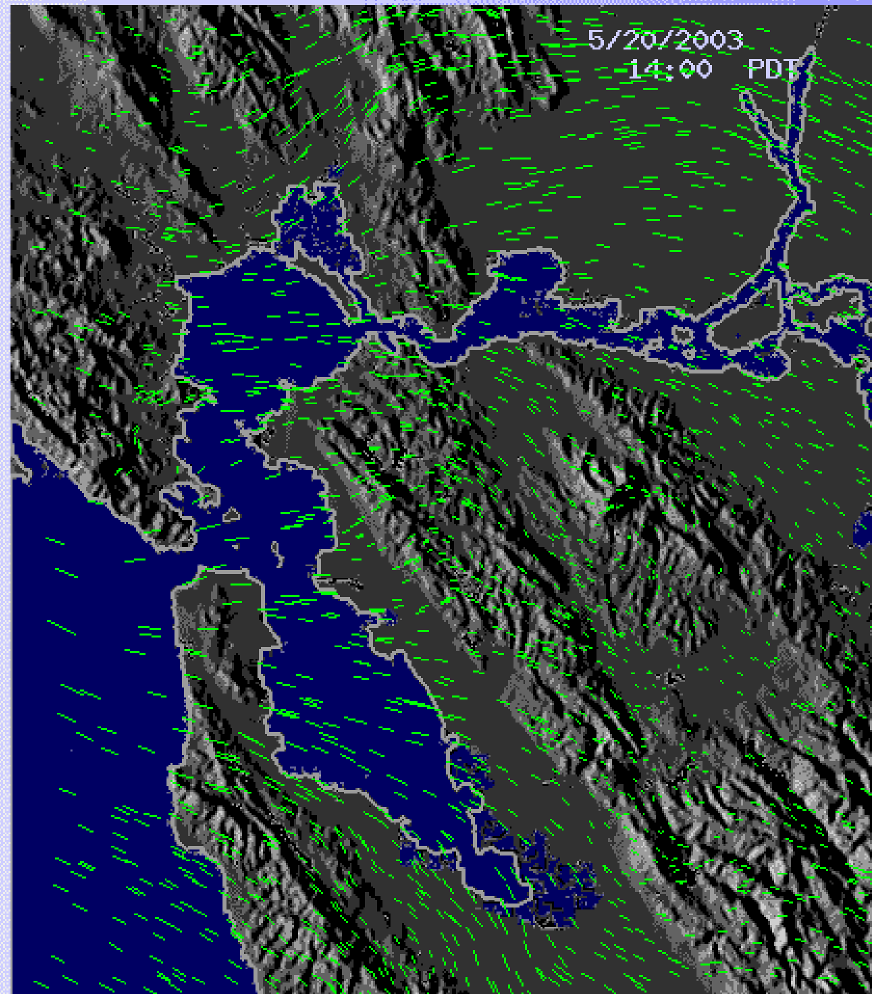
Francis L. Ludwig

**Click to start/stop**

Kudos to Nick Thompson for this applet -- See Description **Below**

Modeled Wind Field Over S.F. Bay

SFPORTS: Tides & Winds, Currents




**Lagrangian Variable:** $\theta = \theta[\vec{X}_o(t_o), \vec{X}(t), t]$

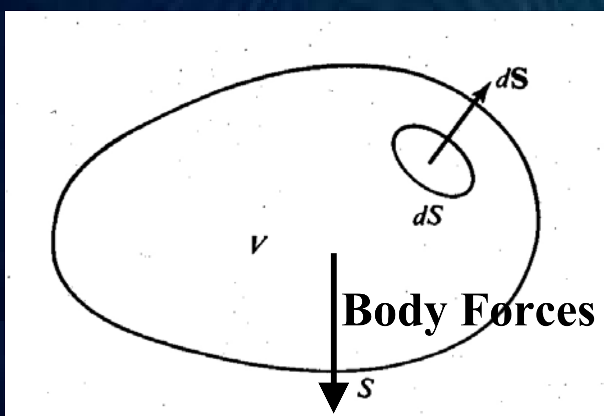
# Physics

**Fluid Dynamics is Lagrangian by nature**

**Eulerian treatments are for convenience**

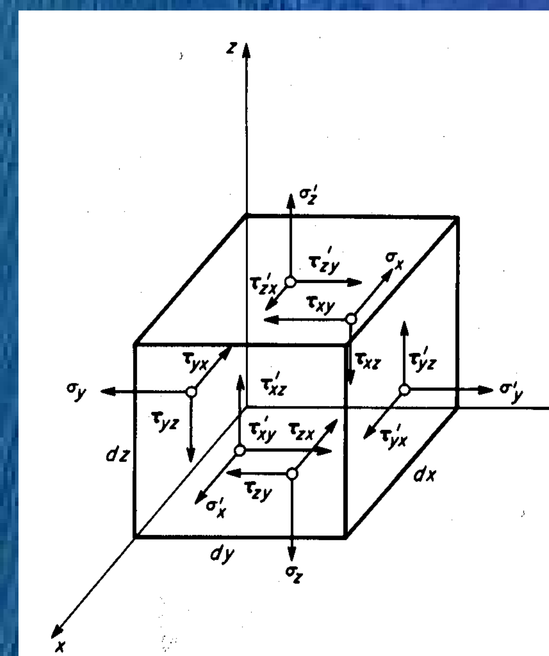
**Second Law of Newton in Fluid Dynamics**

**Lagrangian P.V:**




$$\vec{F} = m\vec{a}$$

**Eulerian P.V:**




$\sum$ **Surface and Body Forces =**

$$\frac{D}{Dt}(Momemtum)$$

# Physics

| Lagrangian Problem: | Eulerian Problem: |
| --- | --- |
| Spilled Oil Slicks | Transport Process |
| Sediment Patches | Pollutants |
| Planktons and Larvae | Salt, Temperature |
| (Biology) | Dissolved Solutes |
| Search and Rescue | |
| **Discrete** | **Continuum** |

# Observations:

**Lagrangian Point of View:**

- Physics is clear
- Discrete particle dynamics
- Measurement difficulties
- Hard to quantify measurements

**Eulerian Point of View:**

- Continuum
- Operational Convenience
- Easy to organize "information"

**Euler-Lagrangian Transformation:**

**Substantial Derivative**

$$\frac{D\theta}{Dt} = \frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}$$

# Some Common Measurement Techniques:

## Lagrangian Reference Frame:

**Lagrangian Variable:** $\theta = \theta[\vec{X}_o(t_o), \vec{X}(t), t]$

**Most flow visualization techniques**

**Dye studies, drifters**

**Long-term path of water ‘mass’**

**Measurement Difficulties, Hard to quantify measurements**

## Eulerian Reference Frame:

**Eulerian Variable:** $\theta = \theta(x, y, z, t)$

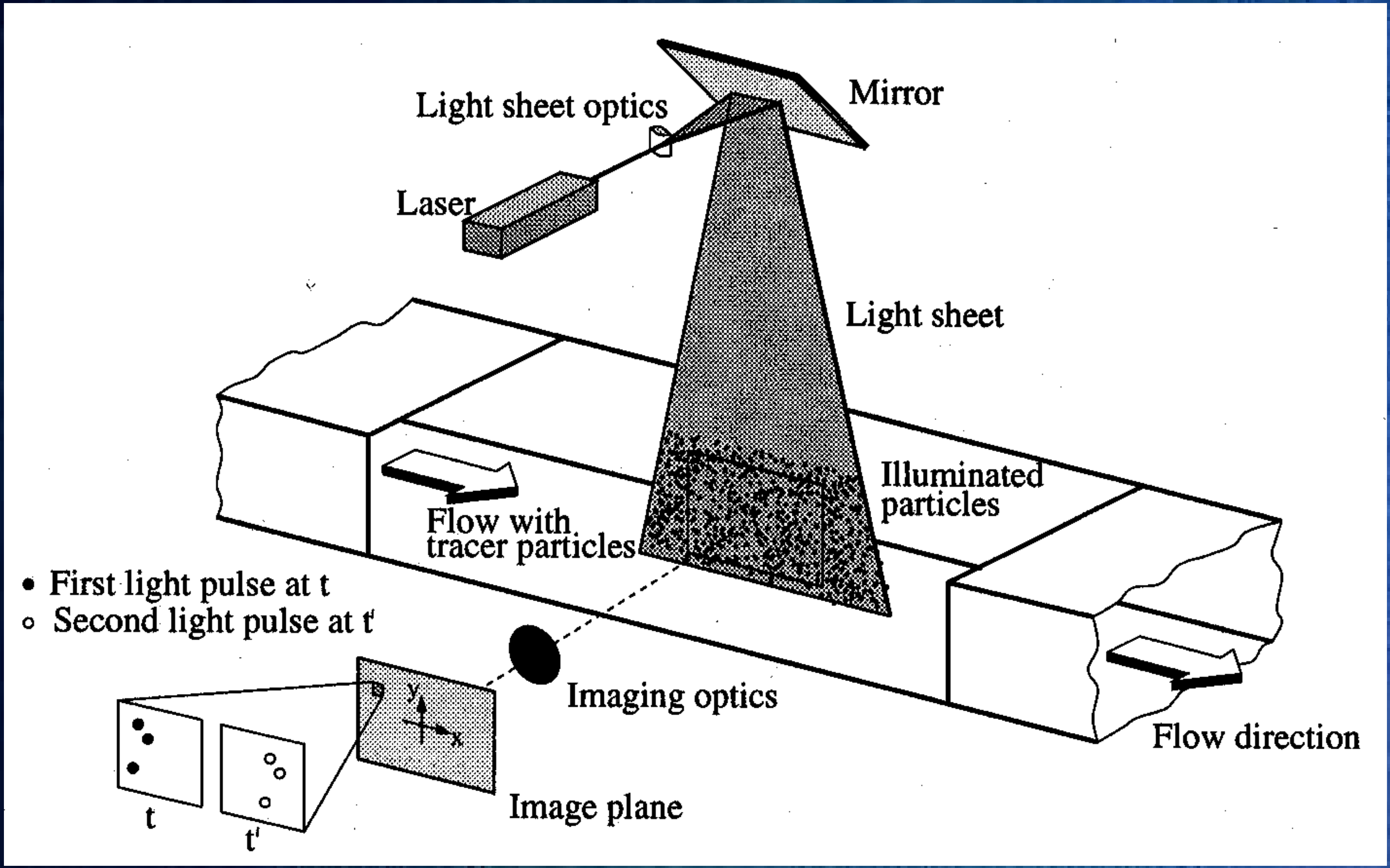
**Fixed Current Meter, CTD moorings**

**Cruising and Profiling ADCP, CTD**

**HF Radar for surface current and waves**

**Operational Convenience, Easy to organize “information”**

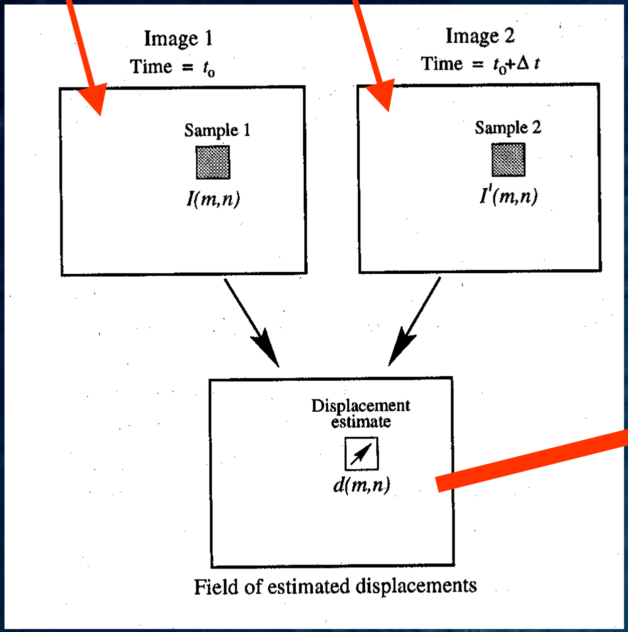
# Combined Eulerian-Lagrangian Measurement Techniques: Particle Image Velocimetry (PIV)

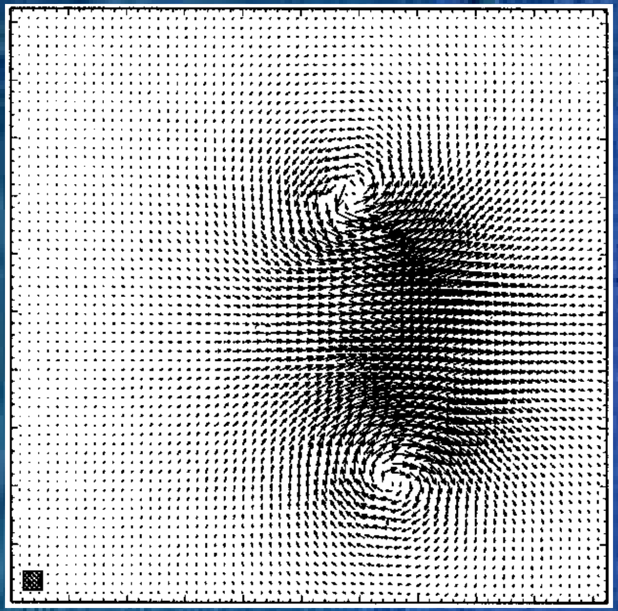



Particle Image Velocimetry by M. Raffel, C. Willert, J. Kompenhans, Springer, 1998.

Lagrangian Observations

Map results to an Eulerian Reference Frame



Estimating displacements by cross-correlations

PIV has been successfully extended to include multi-cameras, to three-dimensional flows, turbulence, …., etc.

Observation: The technique is mature in laboratory applications!

Is there room for applications of Particle Image Velocimetry (PIV) in geophysical & environmental fluid flows?

Have you noticed that weather forecasts are more accurate?

Difference? Temporal and spatial scales

Some applications in rivers

We have limited success in field applications

Challenge: Applications of PIV in environmental flow studies?

# Numerical Methods

**Lagrangian Point of View:**

**Clear Physics**

**Difficulties to quantify measurements**

**Eulerian Point of View:**

**Continuum, Operational Convenience**

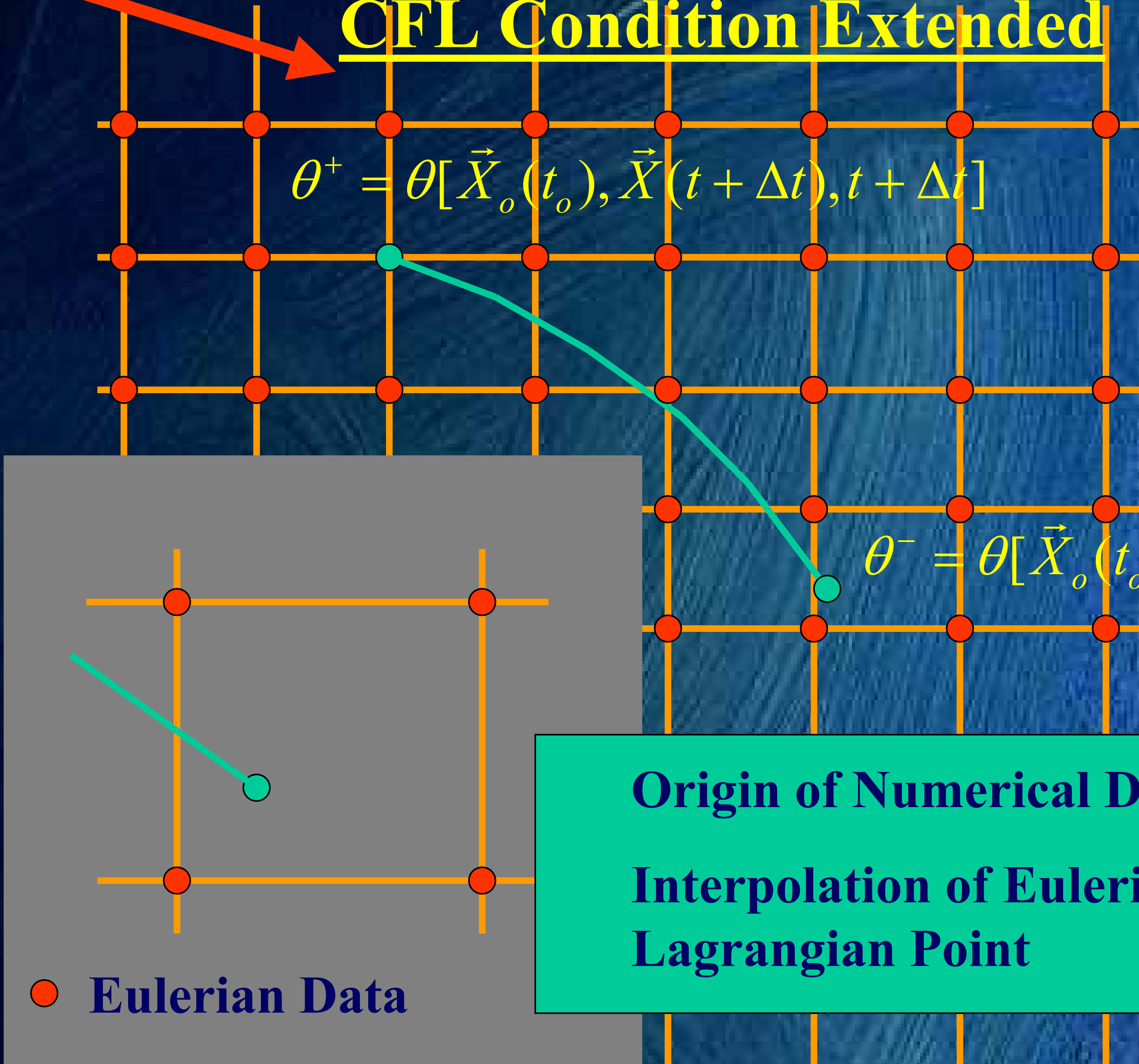
**Easy to organize "information"**

**Substantial Derivative: Euler-Lagrangian Transformation**

$$\frac{D\theta}{Dt} = \frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}$$

$$\frac{D\theta}{Dt} = \frac{\theta^{+} - \theta^{-}}{\Delta t} = \frac{\theta[X(t_o + \Delta t)] - \theta[X(t_0)]}{\Delta t}$$

# Eulerian-Lagrangian Approach: CFL Condition Extended




$$\theta^{+} = \theta[\vec{X}_o(t_o), \vec{X}(t+\Delta t), t+\Delta t]$$

$$\theta^{-} = \theta[\vec{X}_o(t_o), \vec{X}(t), t]$$

**Origin of Numerical Dispersion:**

**Interpolation of Eulerian Data to Lagrangian Point**

## Summary:

**Lagrangian Point of View:**

- Clear Physics
- Discrete Labeled Water Parcel
- Measurement Difficulties (Easier numerically)
- Hard to quantify measurements!

**Eulerian Point of View:**

- Operational Convenience
- Easy to organize "information"
- Needed "information" are populated on an Eulerian Model Grid points (database)

## Consider:

## Eulerian-Lagrangian Method (ELM)

# Recent Italian Connection
# Numerical Modeling
## Collaborations with Vincenzo Casulli

**Cheng, R.T., and Casulli, V., 1982, On Lagrangian residual currents with application in South San Francisco Bay, CA, Water Resources Research, v. 18, No. 6, p. 1652-1662.**

# Recent Italian Connection
# Numerical Modeling

## Collaborations with Vincenzo Casulli

## The TRIM Family of Models
## From TRIM to UnTRIM

- Solution of Shallow Water Equations
- Transient, Multi-Dimensional (3D, 2D, 1D)
- Simultaneous Solution of Transport Variables
- Semi-implicit Finite-Difference Method
- Boundary Fitting Unstructured Grid Mesh

# General Viewpoint of Numerical Modeling of Environmental Flows

## Scales: Physical Properties or Physical Processes

## Spatial and Temporal

# Scales

Need the <u>Right Model</u> to represent the proper physical properties and to resolve the physical processes of the environmental problem

# Formulating the Algorithm for a Numerical Model

**Desirable Properties of a Numerical Model:**

1. Stability
2. Accuracy (Require compromise)
3. Efficiency

**Numerical Algorithm**

**From PDE to Discrete Algebraic System:**

**Spatial discretization:**

Finite difference, Finite Element, Finite Volume

**Temporal discretization:**

Explicit scheme, Implicit scheme, Semi-implicit

# Numerical Foundation of TRIM (Background)

**Casulli, V., 1990, Semi-implicit Finite-difference Methods for the Two-dimensional Shallow Water Equations, J. Comput. Phys., V. 86, p. 56-74.**

**Desirable Properties of a Numerical Model:**
**1. Stability 2. Accuracy 3. Efficiency**
**(Compromise)**

**Stability Analysis:** **Gravity wave terms and velocities in Continuity Eq. control the numerical stability**

**Method of Solution:**

1. **Treat those terms implicitly, and the remaining terms explicitly.**

2. **Substituting momentum Eqs. into continuity Eq., resulting a matrix equation that determines the water surface of the entire domain.**

# 2D Depth-Averaged Shallow Water Equations

**Continuity Eq.:**

$$\frac{\partial \varsigma}{\partial t} + \frac{\partial[(h+\varsigma)U]}{\partial x} + \frac{\partial[(h+\varsigma)V]}{\partial y} = 0$$

**X-Momentum Eq.:**

$$\frac{DU}{Dt} - fV = -g\frac{\partial \varsigma}{\partial x} + \frac{1}{\rho_o(h+\zeta)}(\tau_x^w - \tau_x^b) + A_h \nabla^2 \mathrm{U} - \frac{g}{2\rho_o}(h+\varsigma)\frac{\partial \rho}{\partial x}$$

**Y-Momentum Eq.:**

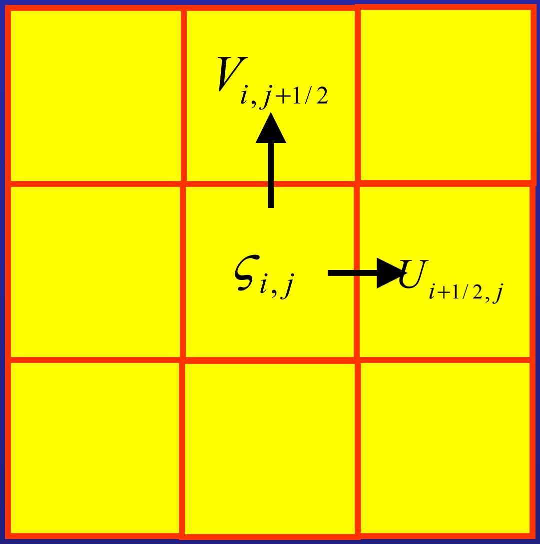
$$\frac{DV}{Dt} + fU = -g\frac{\partial \varsigma}{\partial y} + \frac{1}{\rho_o(h+\zeta)}(\tau_y^w - \tau_y^b) + A_h \nabla^2 \mathrm{V} - \frac{g}{2\rho_o}(h+\varsigma)\frac{\partial \rho}{\partial y}$$

**Eulerian-Lagrangian Method (ELM) => Stability (von Neumann)**

## X-Momentum Eq.:

$$\frac{DU}{Dt} - fV = -g\frac{\partial \varsigma}{\partial x} + \frac{1}{\rho_o(h+\zeta)}(\tau_x^w - \tau_x^b) + A_h\nabla^2 \mathrm{U} - \frac{g}{2\rho_o}(h+\varsigma)\frac{\partial \rho}{\partial x}$$

**Semi-implicit FD: Algebraic Eq. of** $\varsigma_{i,j}^{n+1}, U_{i+1/2,j}^{n+1}, \varsigma_{i+1,j}^{n+1}$




## Y-Momentum Eq.:

$$\frac{DV}{Dt} + fU = -g\frac{\partial \varsigma}{\partial y} + \frac{1}{\rho_o(h+\zeta)}(\tau_y^w - \tau_y^b) + A_h\nabla^2 \mathrm{V} - \frac{g}{2\rho_o}(h+\varsigma)\frac{\partial \rho}{\partial y}$$

**Semi-implicit FD: Algebraic Eq. of** $\varsigma_{i,j}^{n+1}, V_{i,j+1/2}^{n+1}, \varsigma_{i,j+1}^{n+1}$

# Substituting the momemtum Equations into

**Continuity Eq.:** $$\frac{\partial \varsigma}{\partial t} + \frac{\partial[(h+\varsigma)U]}{\partial x} + \frac{\partial[(h+\varsigma)V]}{\partial y} = 0$$

$$(1 + A_{i+1,j} + B_{i-1,j} + C_{i,j+1} + D_{i,j-1})\varsigma_{i,j}^{n+1}$$
$$-A_{i+1,j}\varsigma_{i+1,j}^{n+1} - B_{i-1,j}\varsigma_{i-1,j}^{n+1} - C_{i,j+1}\varsigma_{i,j+1}^{n+1} - D_{i,j-1}\varsigma_{i,j-1}^{n+1} = E_{i,j}^{n}$$

**With all coefficients are positive.**

**The governing matrix equation is symmetric, diagonally dominant, and positive definite. Numerical solution is achieved by a preconditioned conjugate gradient method.**

# Some Numerical Properties

- Convective terms- Eulerian-Lagrangian method
- Gravity wave terms - unconditionally stable
- Discretized equation - properly accounts for positive and zero depths
- Wetting and drying of cells are treated correctly
- TRIM2D successfully implemented to reproduce sharp hydrographs of riverine flows and for estuaries
- The model is robust and efficient

TRIM_2D: Extensive applications in San Francisco Bay

Cheng, R. T., V. Casulli, and J. W. Gartner, 1993, Tidal, residual, intertidal mudflat (TRIM) model and its applications to San Francisco Bay, California, Estuarine, Coastal, and Shelf Science, Vol. 36, p. 235-280.

## What does TRIM model stand for?

TRIM stands for Tidal, Residual, Inter-tidal Mudflat

TRIM also implies simple and elegant in numerical algorithm and model code, a goal that we are striving for!

# From TRIM Series of Models to UnTRIM

# Systematic Development of TRIM Models:

## TRIM_3D: Applications in San Francisco Bay and others

**Casulli, V. and R. T. Cheng, 1992, Inter. J. for Numer. Methods in Fluids**

**Casulli, V. and E. Cattani, 1994, Comput. Math. Appl., Stability, accuracy and efficiency analysis of TRIM_3D, $\theta$-method for time-difference**

**Cheng, R. T. and V. Casulli, 1996, Modeling the Periodic Stratification and Gravitational Circulation in San Francisco Bay, ECM-4.**

## TRIM_3D: Non-hydrostatic

**Casulli, V. and G. S. Stelling, 1996, ECM-4**

**Casulli, V. and G. S. Stelling, 1998, ASCE, J. of Hydr. Eng**

## UnTRIM model:

**Casulli, V. and P. Zanolli, 1998, A Three-dimensional Semi-implicit Algorithm for Environmental Flows on Unstructured Grids, Proc. of Conf. On Num. Methods for Fluid Dynamics, University of Oxford.**

# Extension to Unstructured Grid Model -- UnTRIM

**TRIM Modeling Philosophy:**

1. Semi-implicit Finite-Difference Methods
2. Θ-Method for time difference
3. Solutions in Physical Space, regular mesh, no coordinate transformations in x-, y-, or z-directions
4. In complicated domain, refine grid resolution if necessary
5. Pursue computational efficiency and robustness

UnTRIM (Unstructured Grid TRIM model) follows the SAME TRIM modeling philosophy, except the finite-difference cells are boundary fitting unstructured polygons!

# Summary of the UnTRIM Model:

## Governing equations (Hydrostatic Assumption)

**Continuity and Free-surface Equations**

$$Div(\vec{U}) = 0$$

Incompressibility

$$\frac{\partial \varsigma}{\partial t} + \nabla \bullet \left[ \int_{-h}^{\zeta} \vec{V}\, dz \right] = 0$$

Free-surface equation

**Horizontal Momentum Equation in $\vec{N}_j$ direction for velocity $V_j$**

$$\frac{DV_j}{Dt} - f(\nabla \times \vec{V}) \bullet \vec{N}_j = \frac{\partial}{\partial z}\left(\nu_v \frac{\partial}{\partial z} V_j\right) + \nu_h \nabla^2 V_j - g\frac{\partial \zeta}{\partial N_j} - \frac{g}{\rho_o}\frac{\partial}{\partial N_j}\int_z^{\zeta}(\rho - \rho_o)dz'$$

**where $\nabla\times(\ )$ is cross product, $\nabla\bullet(\ )$ is inner product, $\nabla^2(\ )$ is the Laplacian, and $\vec{V}$ is the velocity in the horizontal plane.**

**Transport Equations**

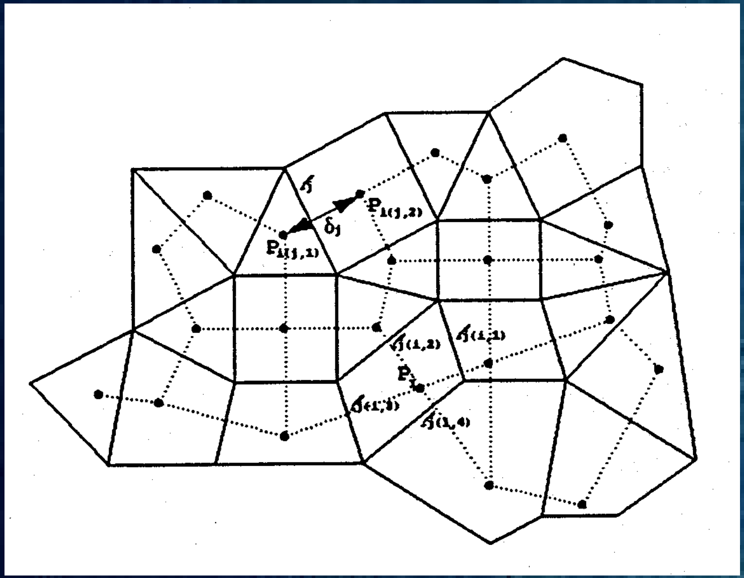
$$\frac{D}{Dt}C_j = \frac{\partial}{\partial z}\left(K_v \frac{\partial}{\partial z} C_j\right) + K_h \nabla^2 C_j \qquad j = 1, 2, 3, \ldots$$

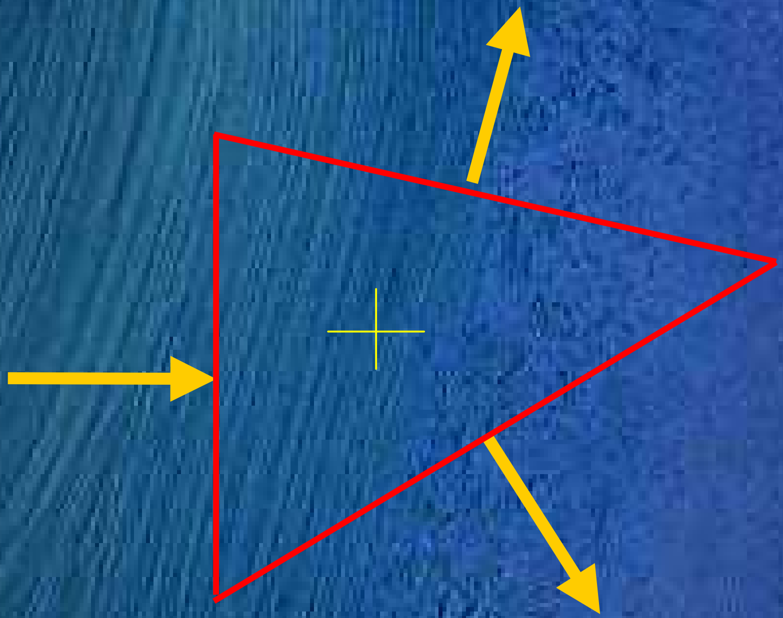
**Lagged one time-step**

**And an equation of State**

1. Semi-implicit finite-difference of momentum Eq. in the normal direction to each face is applied!

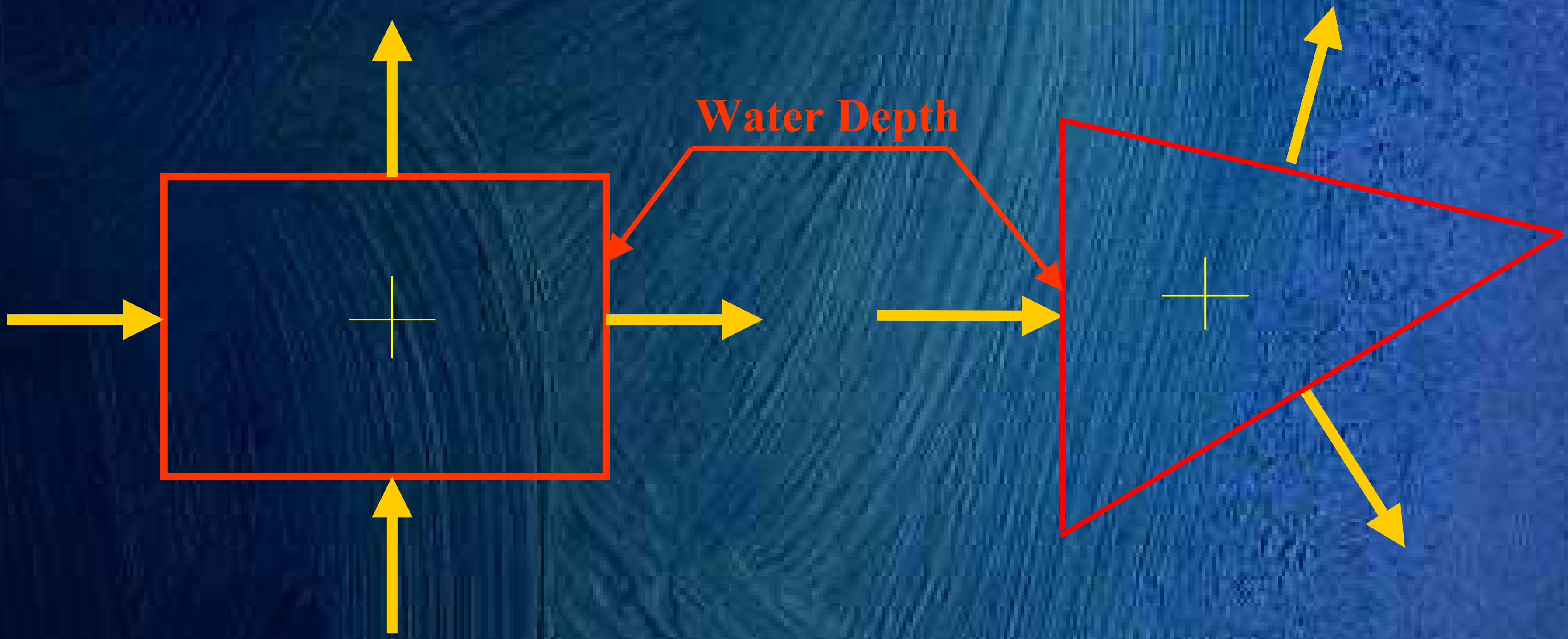
2. Applied the Finite-Volume integration of the free surface equation!
   Local and global conservation of volume is guaranteed!





3. The resultant matrix equation determines the water surface elevation for the entire field.

1. **Semi-implicit finite-difference of momentum Eq. in the normal direction to each face is applied!**
2. **Applied the Finite-Volume integration of the free surface equation!**
   **Local and global conservation of volume is guaranteed!**




3. **The resultant matrix equation determines the water surface elevation for the entire field.**

# Summary of Numerical Algorithm

**Momentum Equation in $\vec{N}_j$ direction for velocity $V_j$ relates**

**$V_j$ and ζ (left) and ζ (right) on <u>each face</u> of a polygon**

**Continuity and Free-surface Equations**

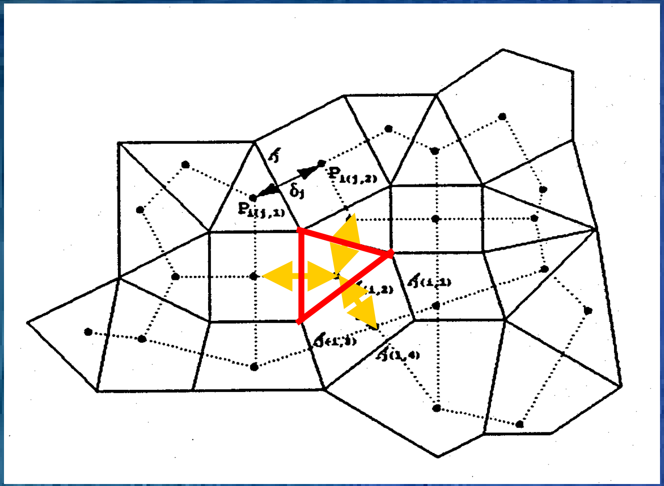
$$Div(\vec{U}) = 0$$

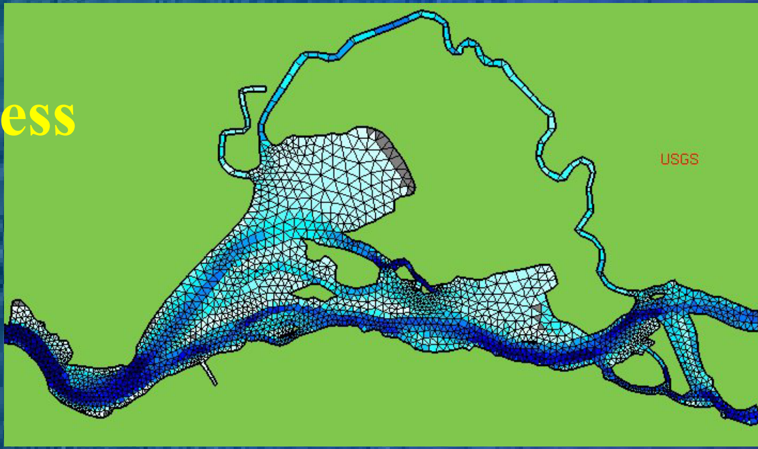
$$\frac{\partial \varsigma}{\partial t} + \nabla \bullet \left[ \int_{-h}^{\varsigma} \vec{V}\, dz \right] = 0 \qquad \Rightarrow \qquad \frac{\partial \varsigma}{\partial t} + \oint \left( \int_{-h}^{\varsigma} \vec{V} dz \right) \bullet d\vec{s} = 0$$

**Finite Volume integration over each polygon =>**
**V's are eliminated giving a Matrix Eq. for ζ**

**The continuity equation and the momentum equations are truly coupled in the solution. No mode splitting is used!**

# Issues of unstructured grids



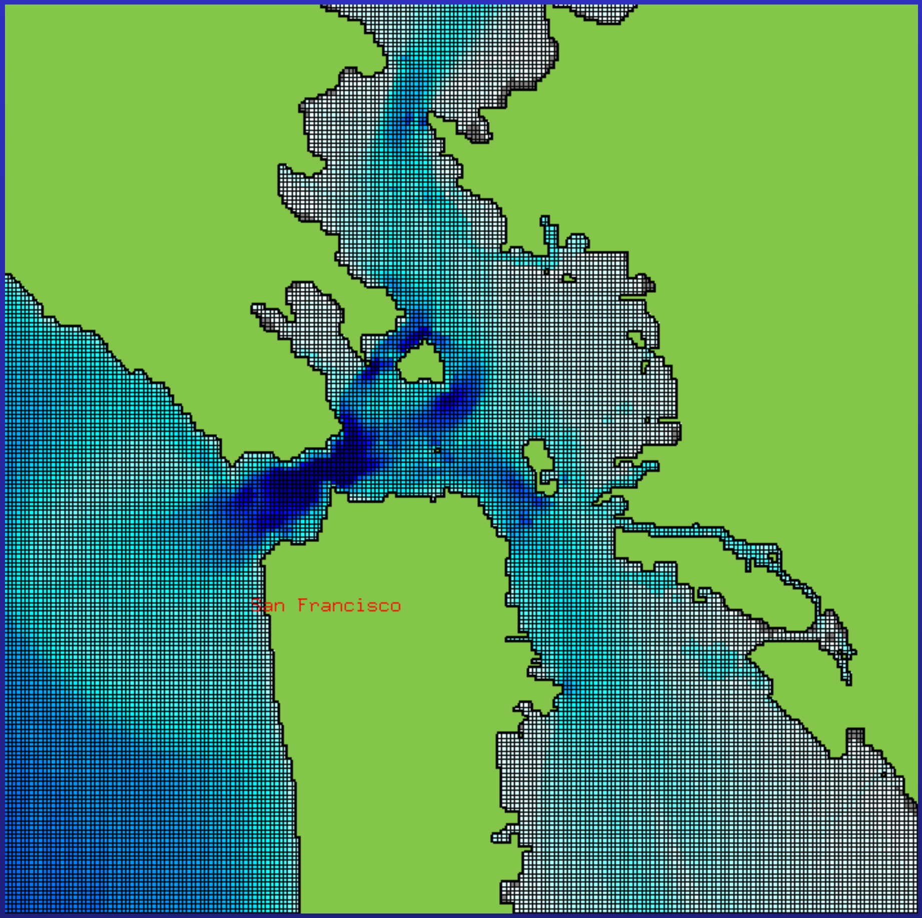



User must define:

1. Number and locations of nodes
2. Polygon number and its relation with nodes (connectivity)
3. Each side is numbered, left and right polygons are defined (connectivity)
4. Center coordinates of each polygon
5. Vertical layers are of constant thickness (variable in z) except the bottom and free-surface; a stack of prisms
6. Water depth and normal velocity are defined on the sides
7. Water elevation is defined at the center of the polygon

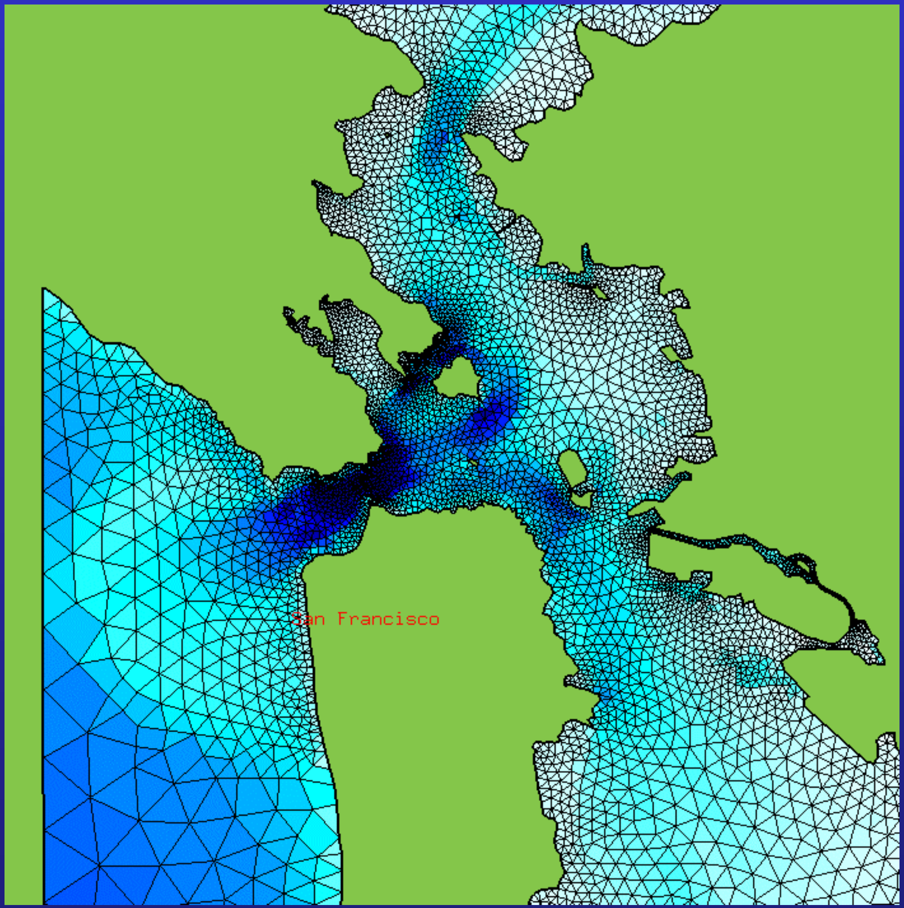
# San Francisco Bay

(All Rectangles)




48506 nodes, 45841 polygons
94374 sides on the top layer
42 layers, 1,160 K faces, Δt = 180
(R= simulation/CPU = 17.7)
on 2.2 GHz PC

(Mixed Polygons)




12682 nodes, 20126 polygons
32827 sides on the top layer
42 layers, 295 K faces, Δt = 180
(R= simulation/CPU = 70)
on 2.2 GHz PC

# A Practical Application Using Italian Tools (UnTRIM)

☺

Wind-Driven Circulation in
Upper Klamath Lake, Oregon

# Modeling Wind-Driven Circulation in Upper Klamath Lake

Ralph T. Cheng*
Jeffrey W. Gartner*
Tamara Wood**
*U. S. Geological Survey, Menlo Park, CA
**U. S. Geological Survey, Portland, OR

I. Background

II. ADCP Deployment and Results

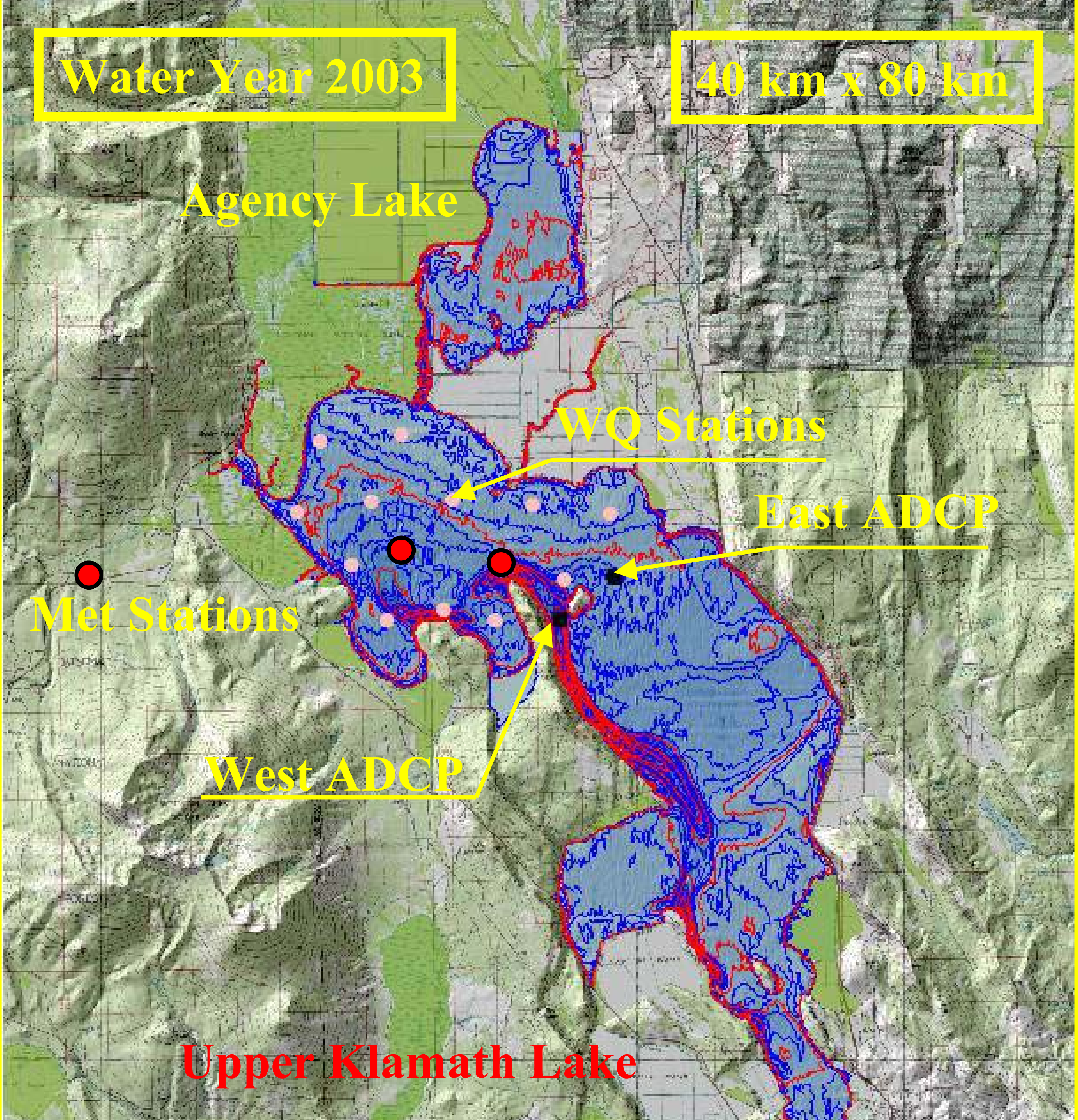
III. Time-series of Wind Observations

IV. Wind-Driven Circulation

V. Reproducing ADCP Observations

VI. Analyze This and Analyze That

VII. Conclusion (Physics Rules!)


Water Year 2003
40 km x 80 km
Agency Lake
WQ Stations
East ADCP
Met Stations
West ADCP
Upper Klamath Lake

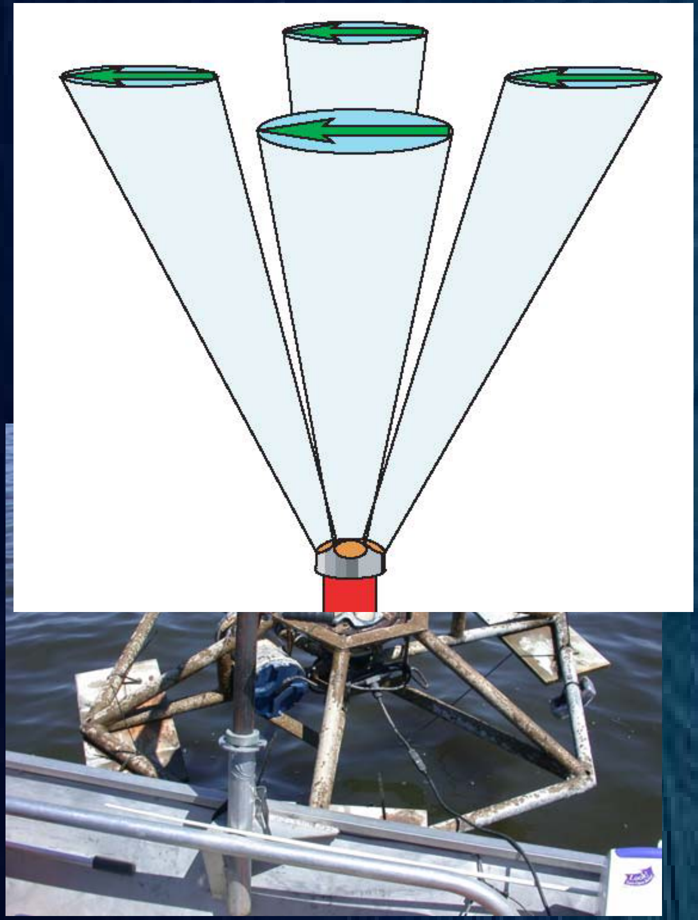
**West ADCP Station:**

**Water depth ~ 8 m**

**Bin size = 0.2 m**

**Sampling rate = 30.0 min**

**Total bins = 34**



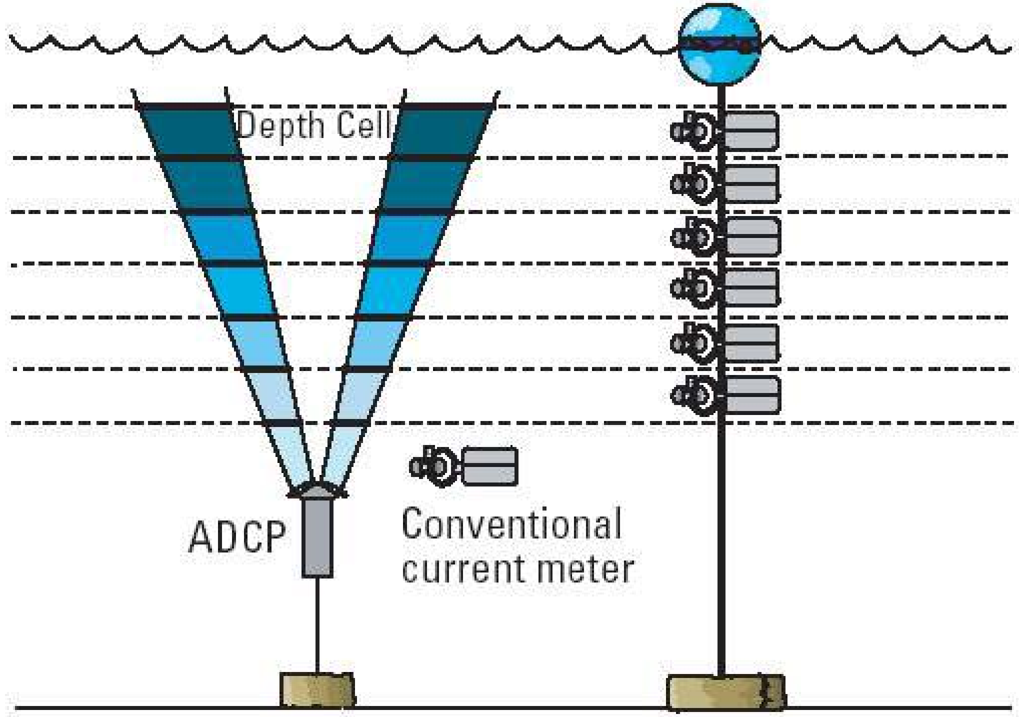



**Figure 1.15.** Analogy of a conventional current-meter string to an acoustic Doppler current profiler (ADCP) profile.

**East ADCP Station:**

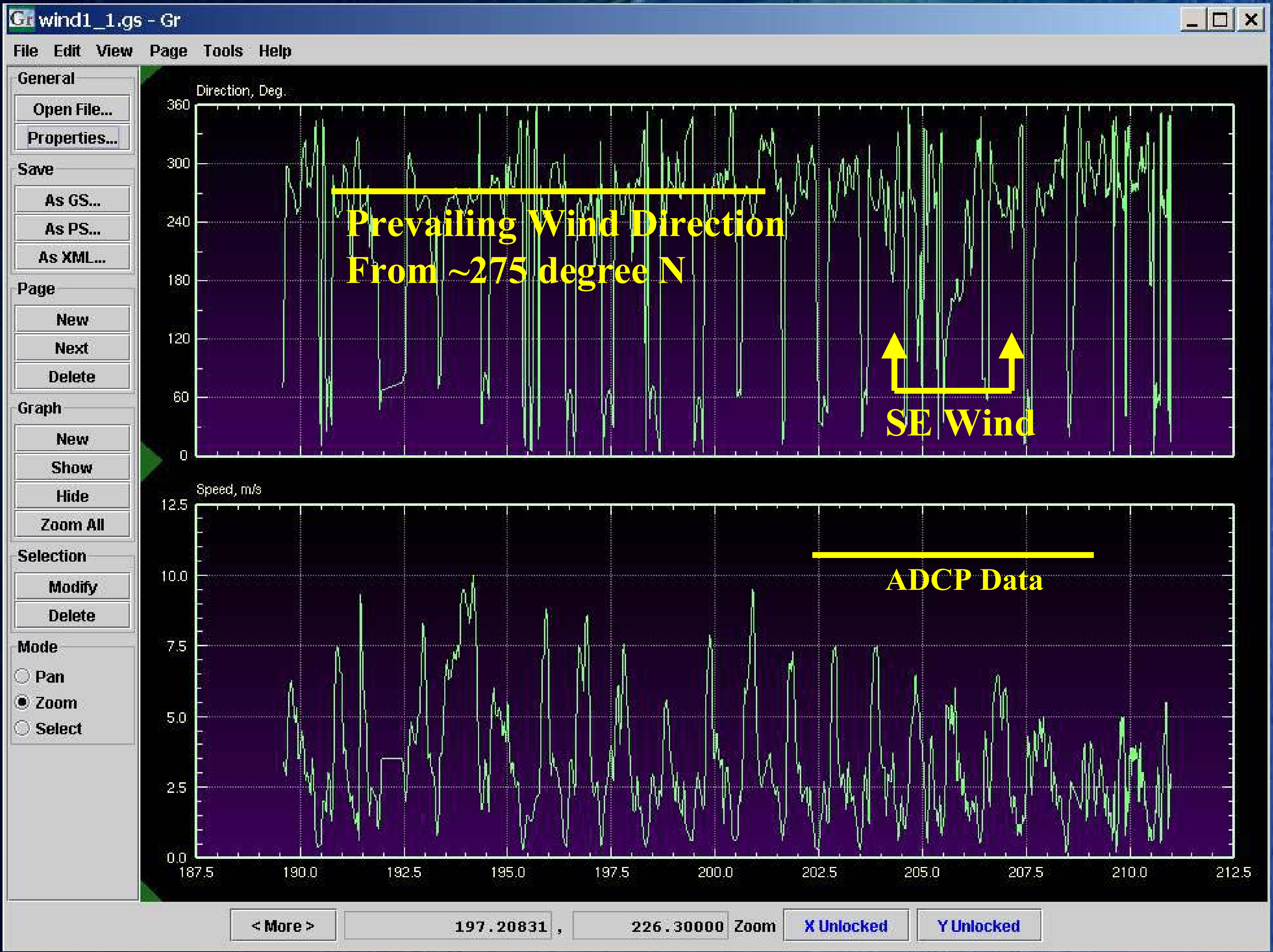
**Water depth ~ 3.5 m**

**Bin size = 0.2 m**

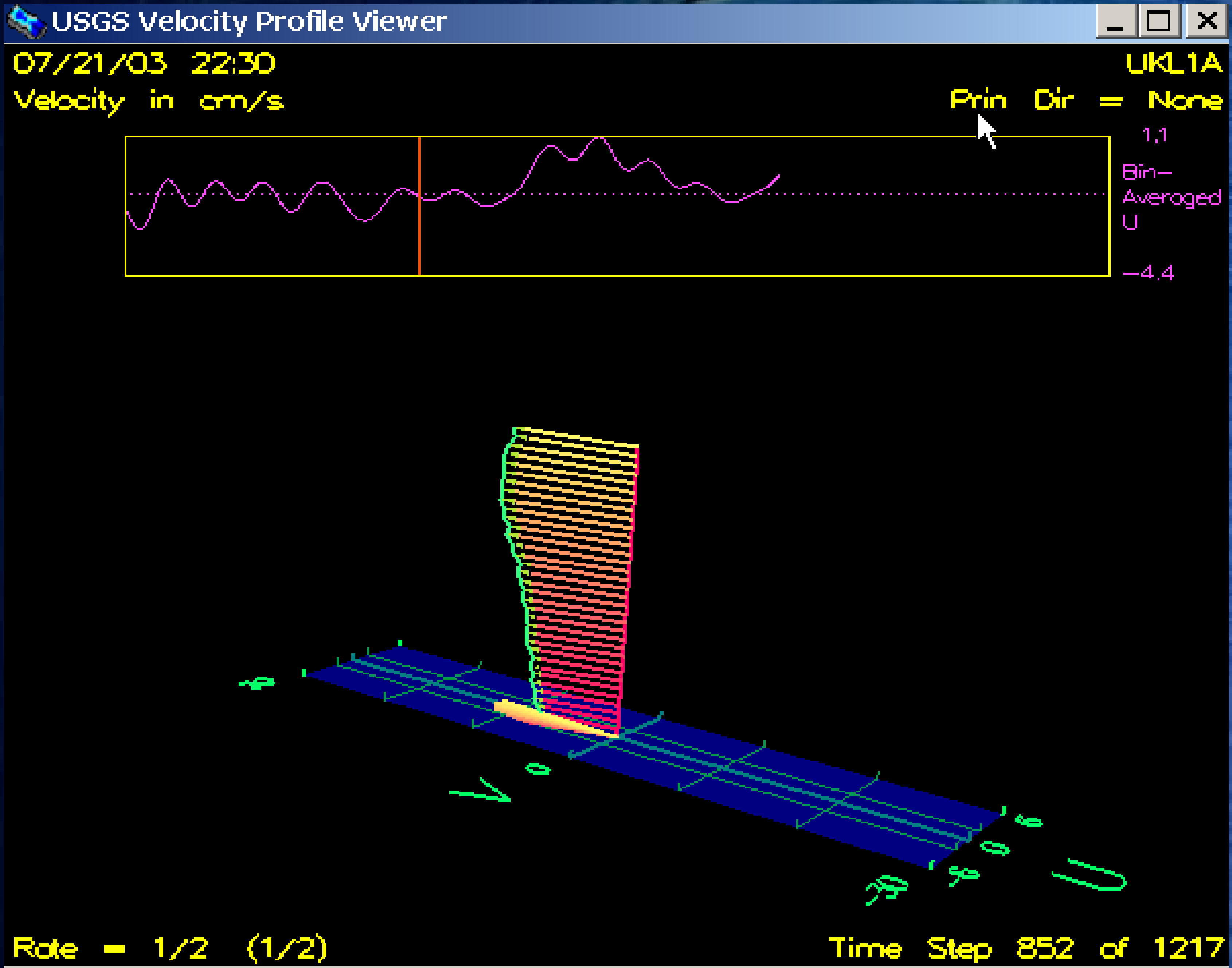
**Sampling rate = 30.0 min**

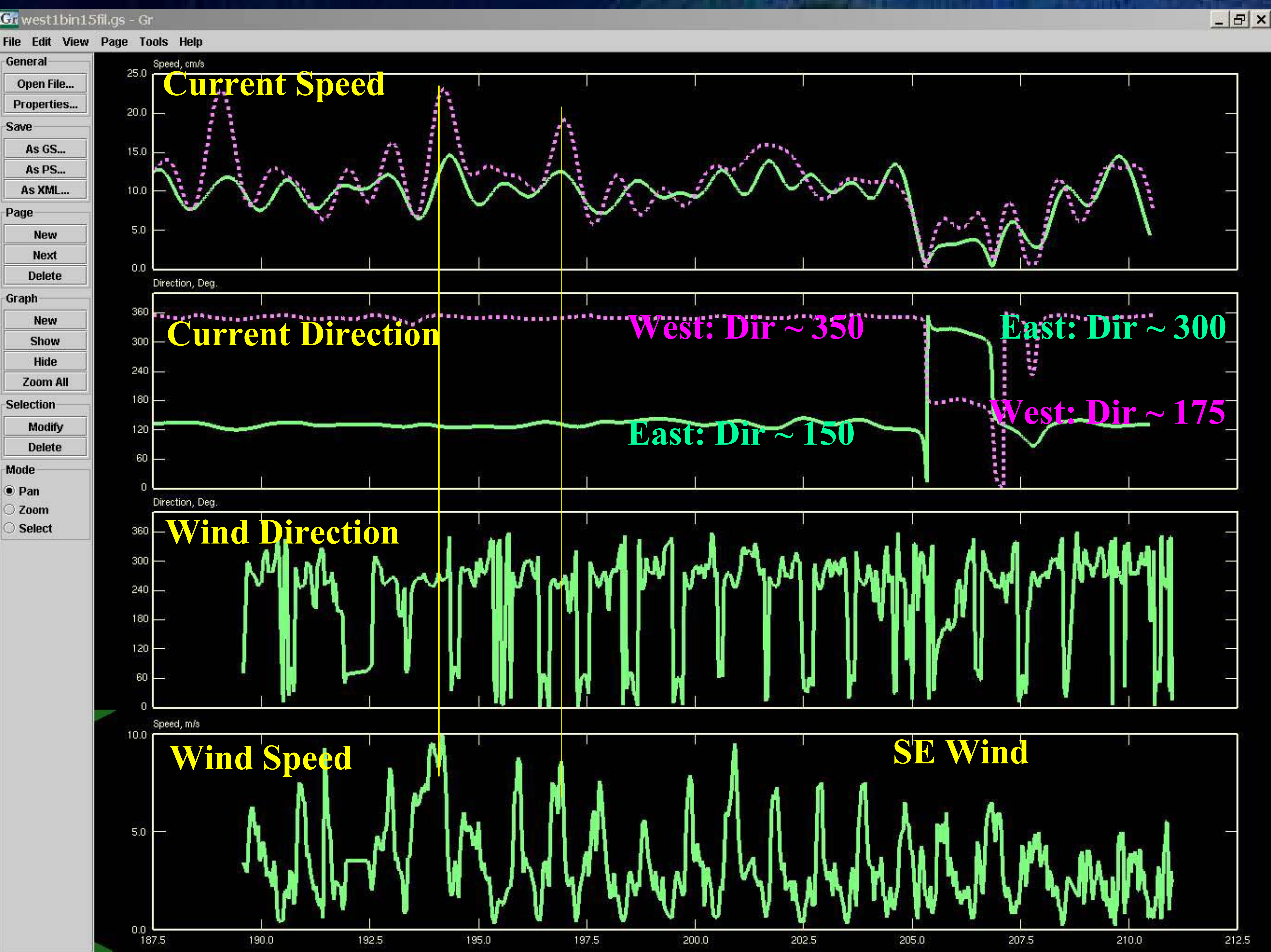
**Total bins = 12**

# Wind Speed and Direction Time-Series



# Filtered 3D ADCP Time-Series




west1bin15fil.gs - Gr
File
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As GS...
As PS...
As XML...
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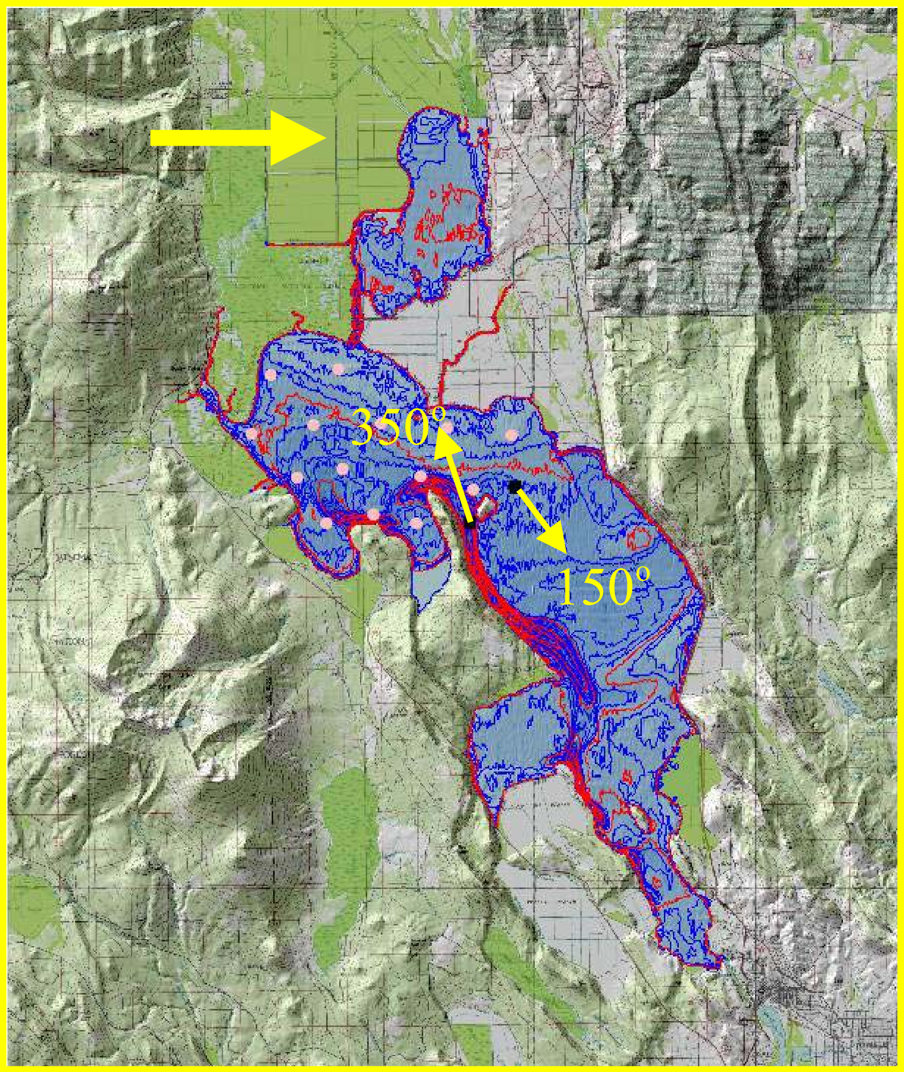
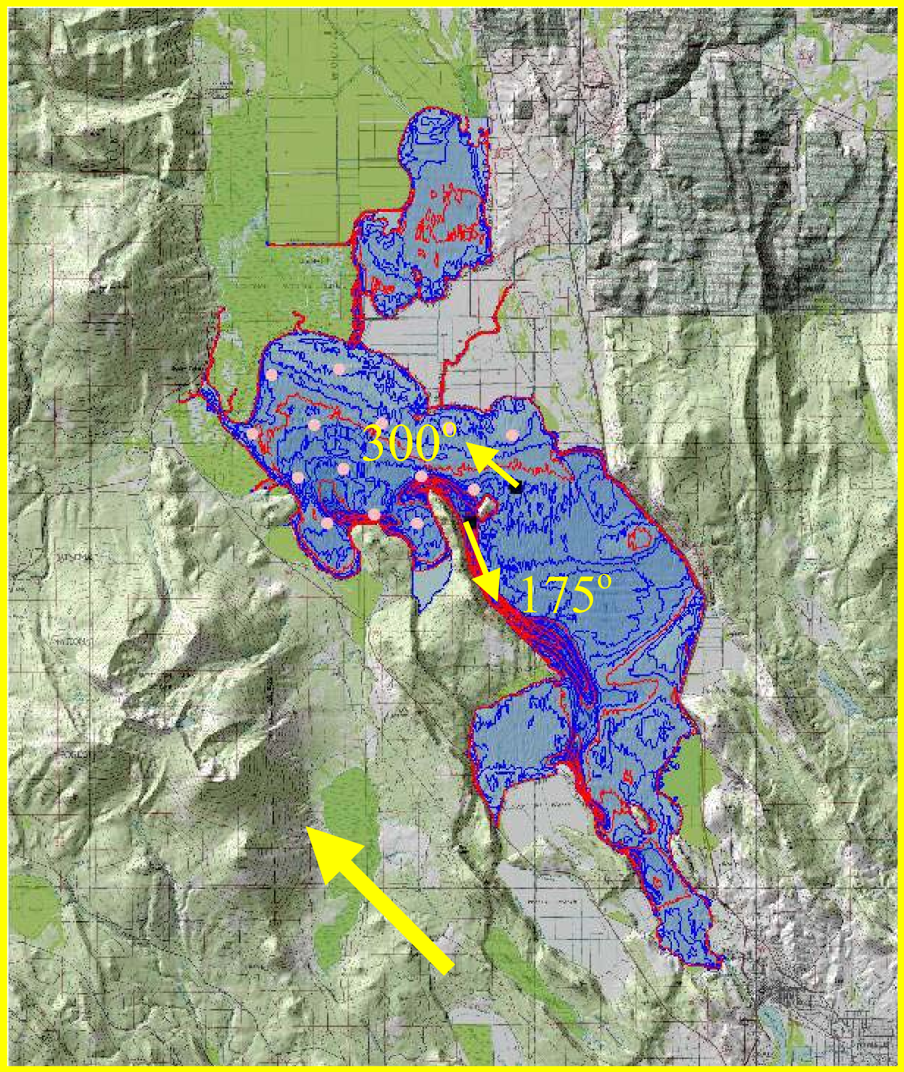
New
Next
Delete
Graph
New
Show
Hide
Zoom All
Selection
Modify
Delete
Mode
Pan
Zoom
Select
Speed, cm/s
Current Speed
Direction, Deg.
Current Direction
West: Dir ~ 350
East: Dir ~ 300
West: Dir ~ 175
East: Dir ~ 150
Direction, Deg.
Wind Direction
Speed, m/s
Wind Speed
SE Wind
187.5
190.0
192.5
195.0
197.5
200.0
202.5
205.0
207.5
210.0
212.5

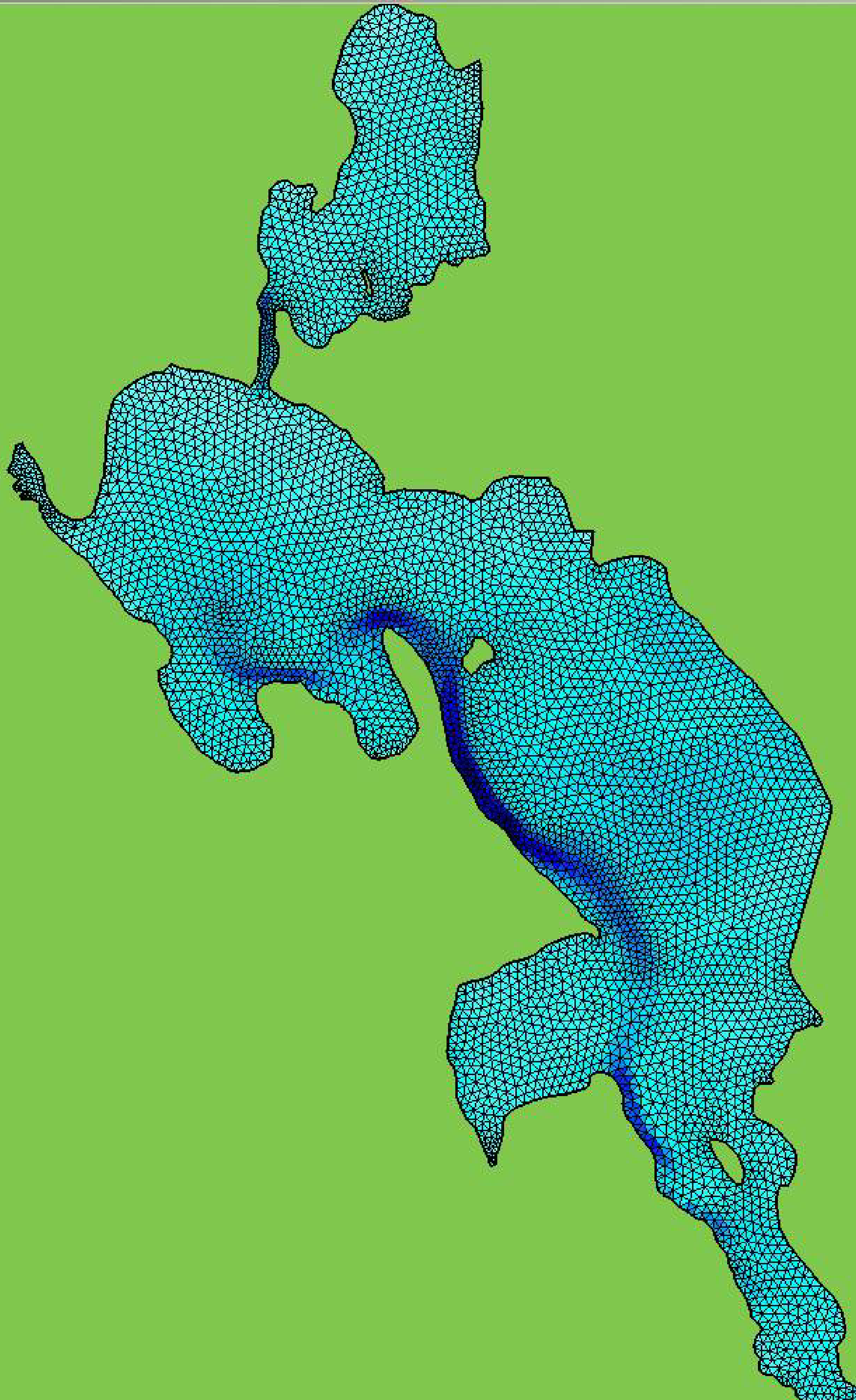
# Synopsis of Wind-driven Circulation




**Prevailing West Wind**




**SE Wind**



**Unstructured Grid Model:**

**Upper Klamath Lake and Agency Lake:**

**nv = 4712**

**ne = 8550**

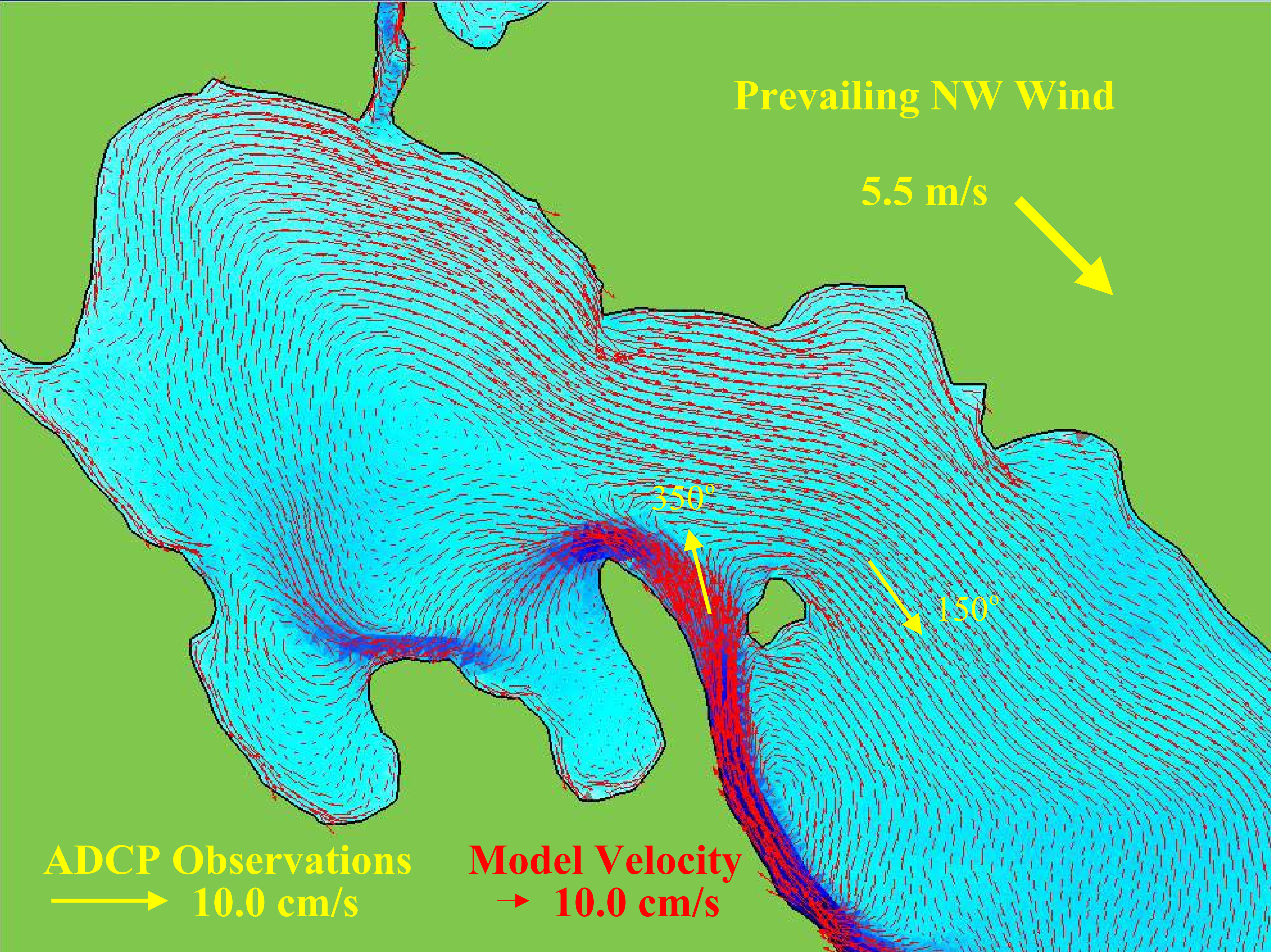
**nk = 22**

**n3s =  82992**

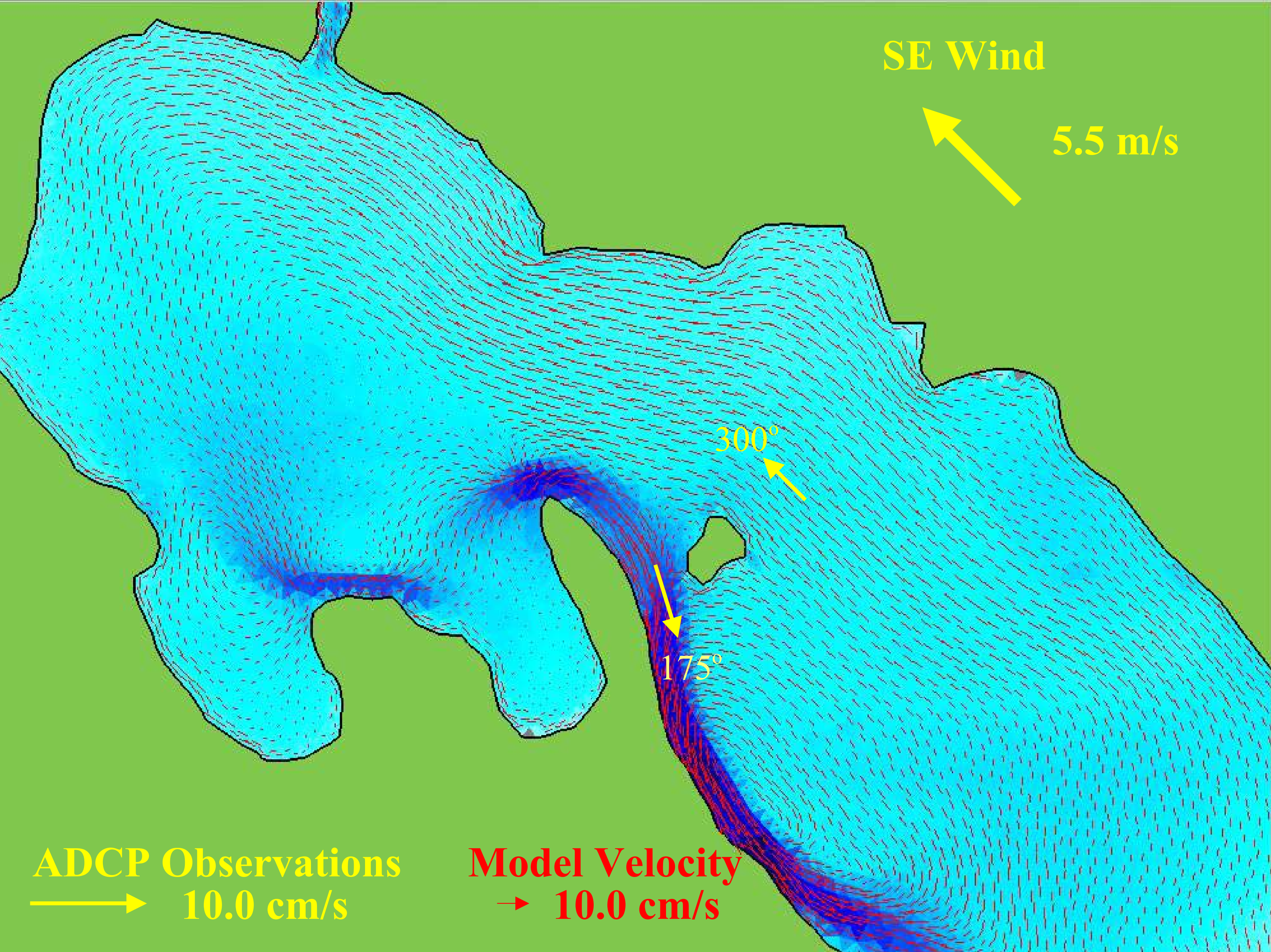
**Side length 40 to 250 m**

**Grids are boundary fitting**

**Fine resolution grids for high spatial variability.**

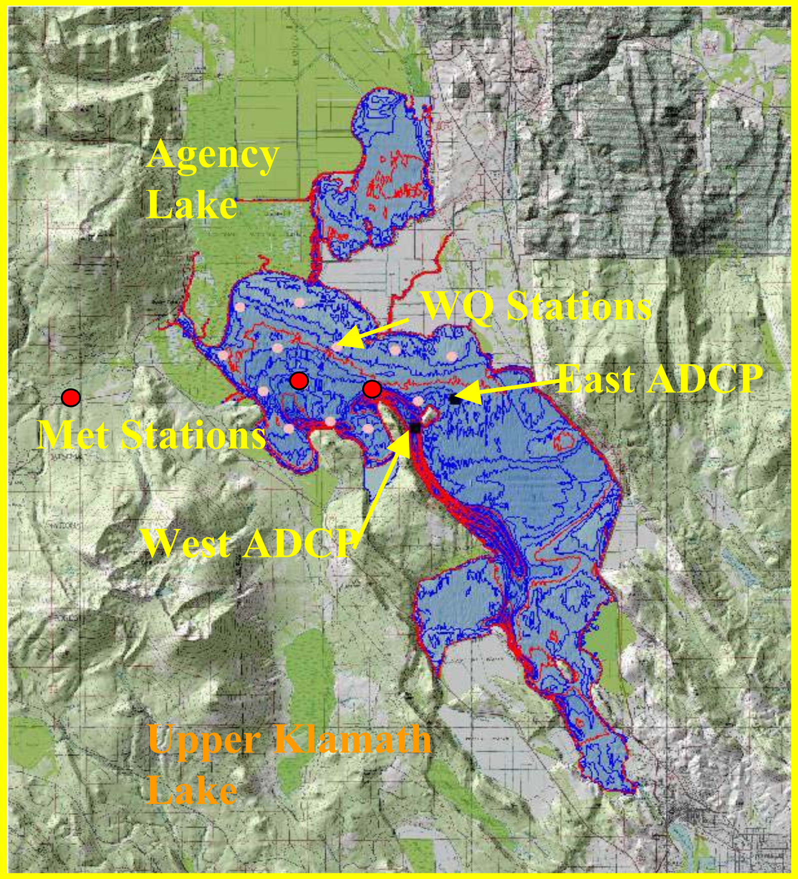


Prevailing NW Wind
5.5 m/s
350°
150°
ADCP Observations
10.0 cm/s
Model Velocity
10.0 cm/s



SE Wind
5.5 m/s
300°
175°
ADCP Observations
10.0 cm/s
Model Velocity
10.0 cm/s

# Simulations based on the observed wind


**Issues with wind time-series:**

1. Magnetic north
2. Data gaps or irregular time intervals

# Field Data:

Observed Wind

Deep ADCP (West)

Shallow ADCP (East)

Williamson River Inflow

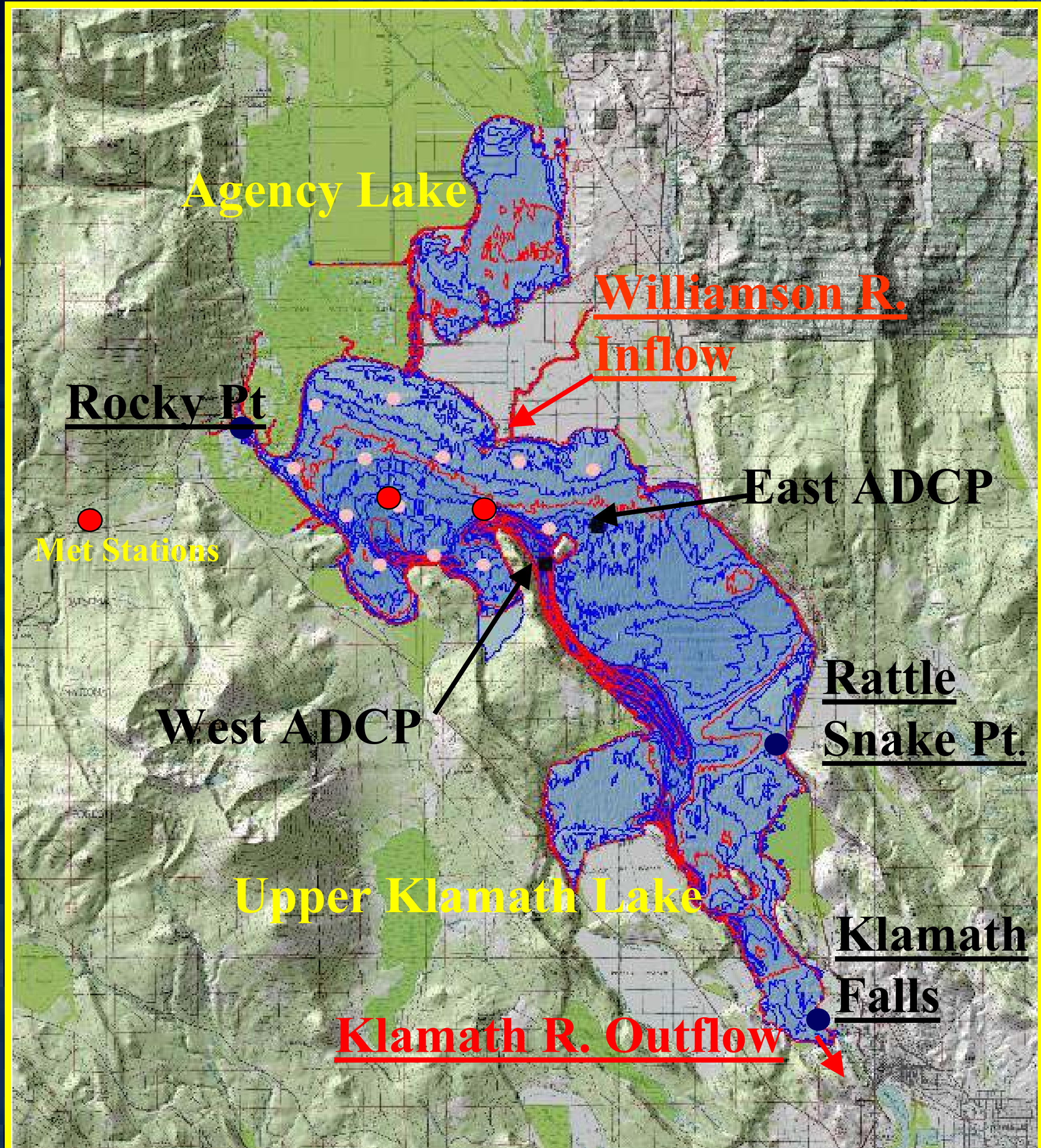
Klamath R. Outflow

Water levels at

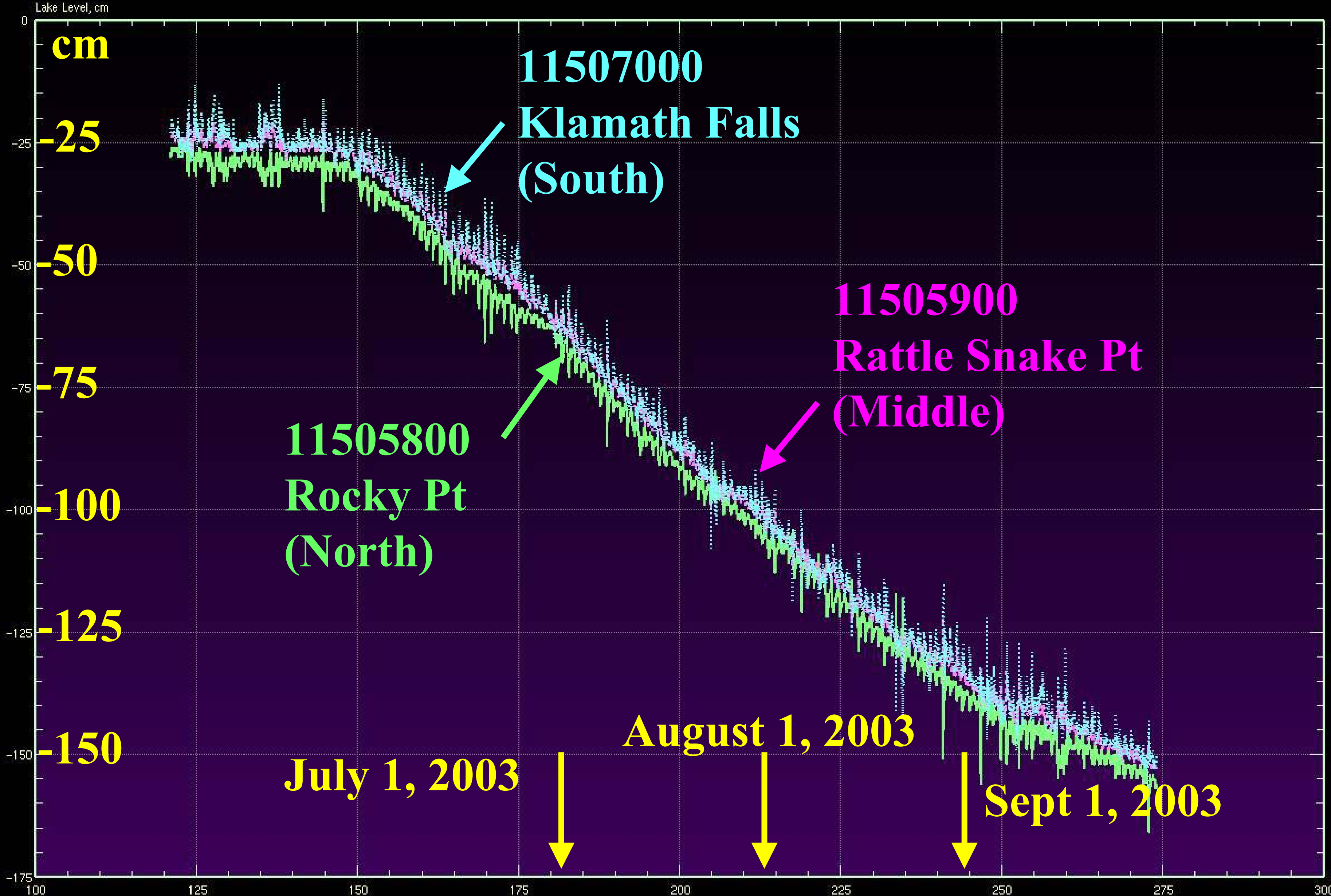
Rocky Pt

Rattle Snake Pt.

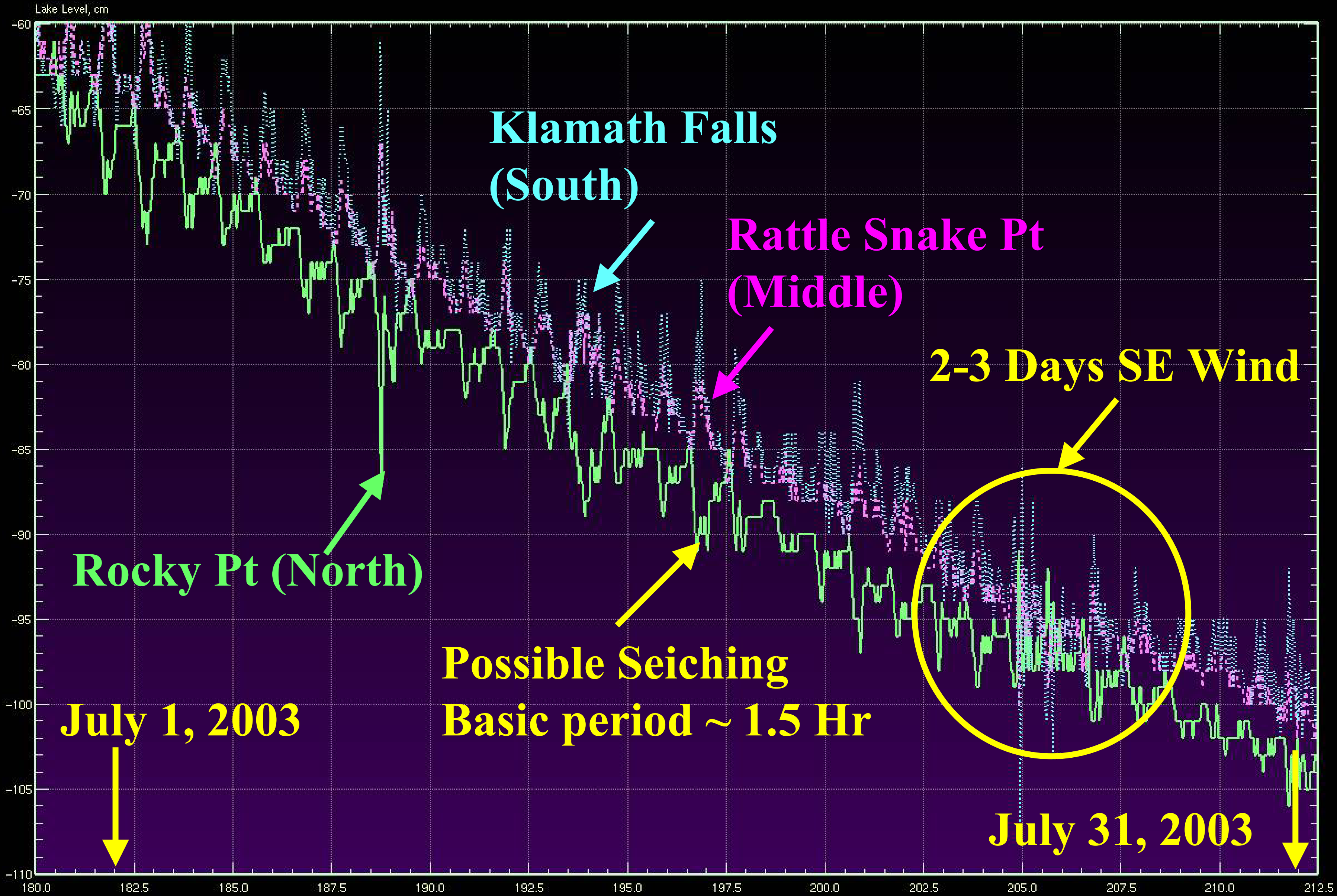
Klamath Falls



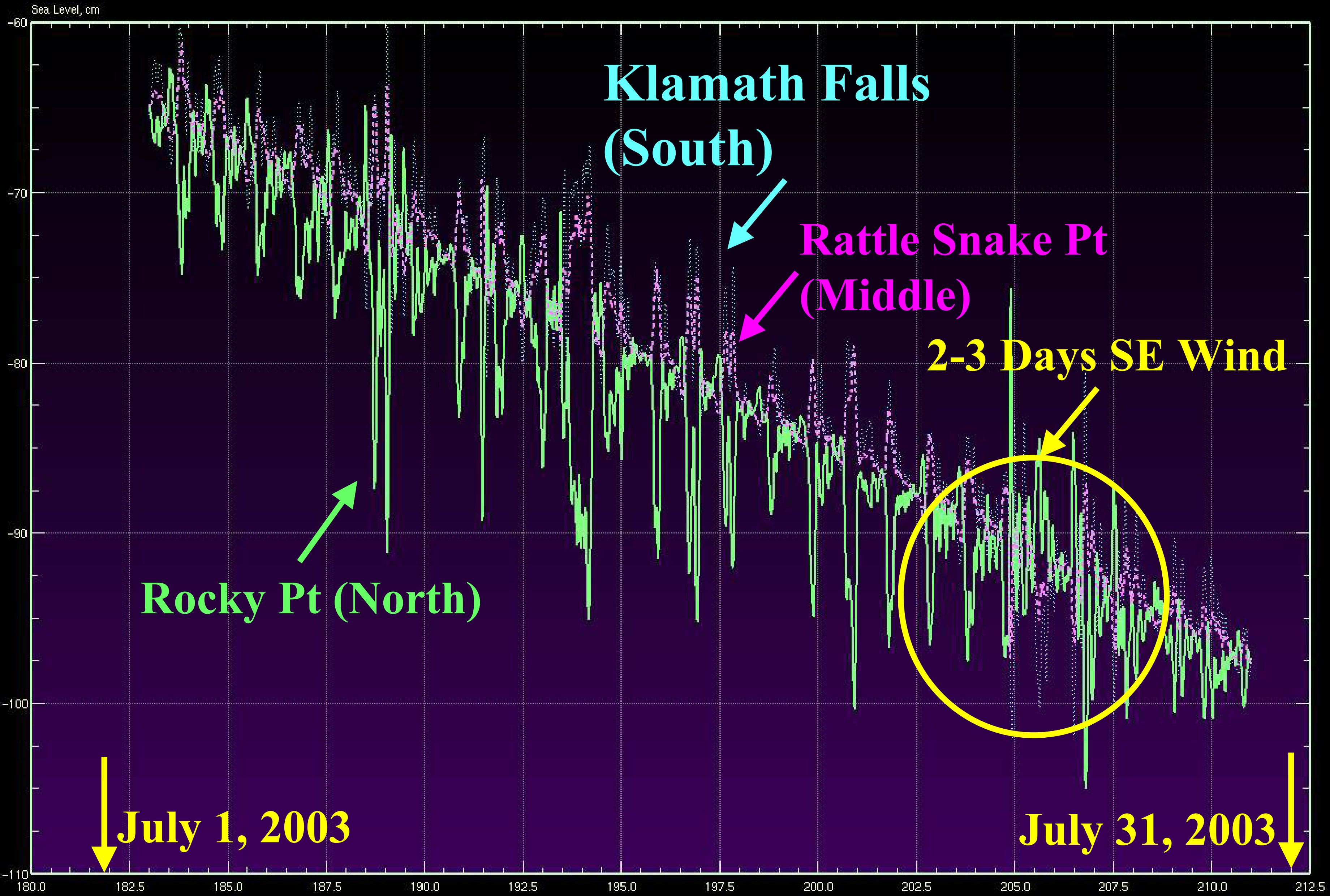
# Water Level Observations Referenced to 4143 ft above sea-level



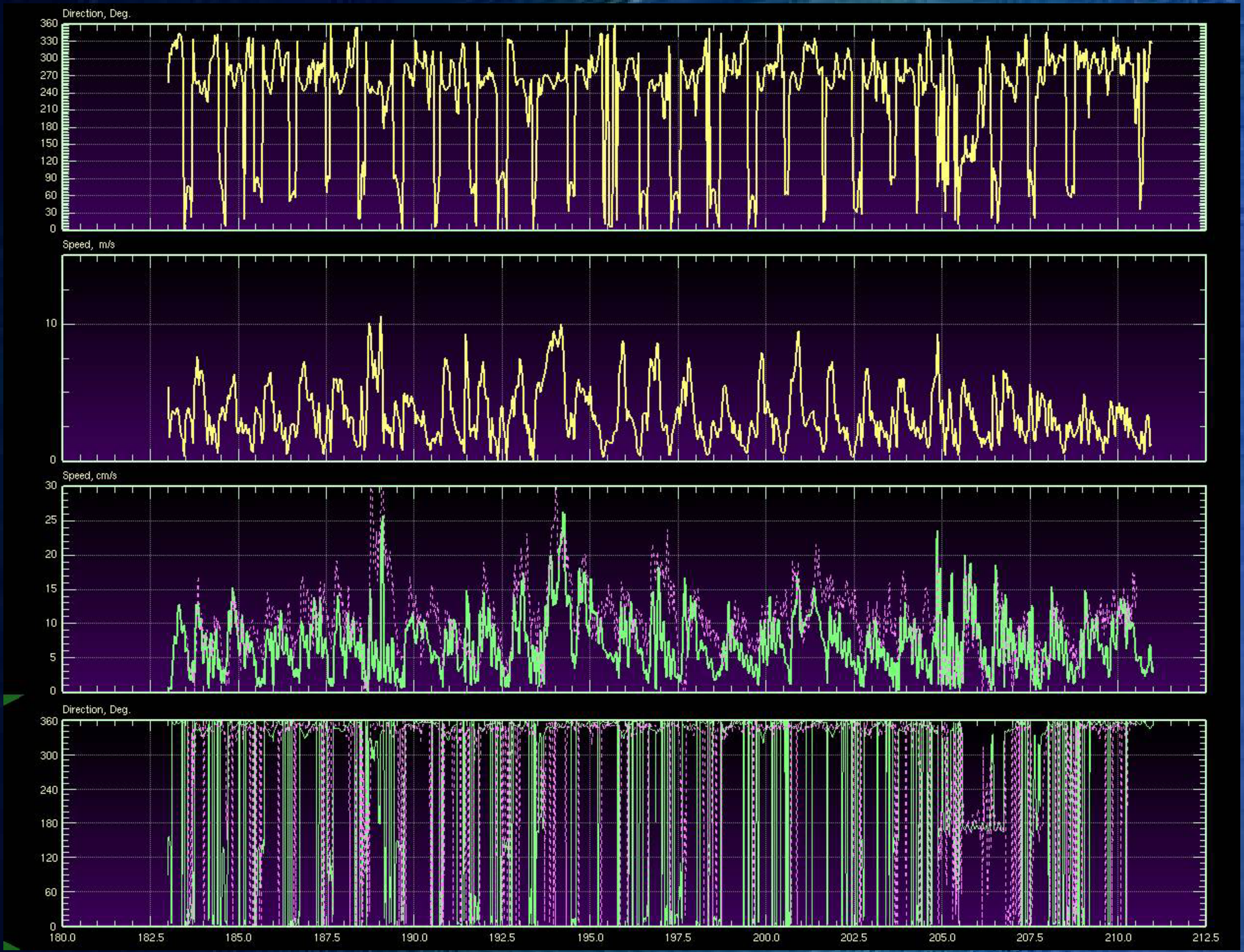
# Water Level Observations Referenced to 4143 ft above sea-level

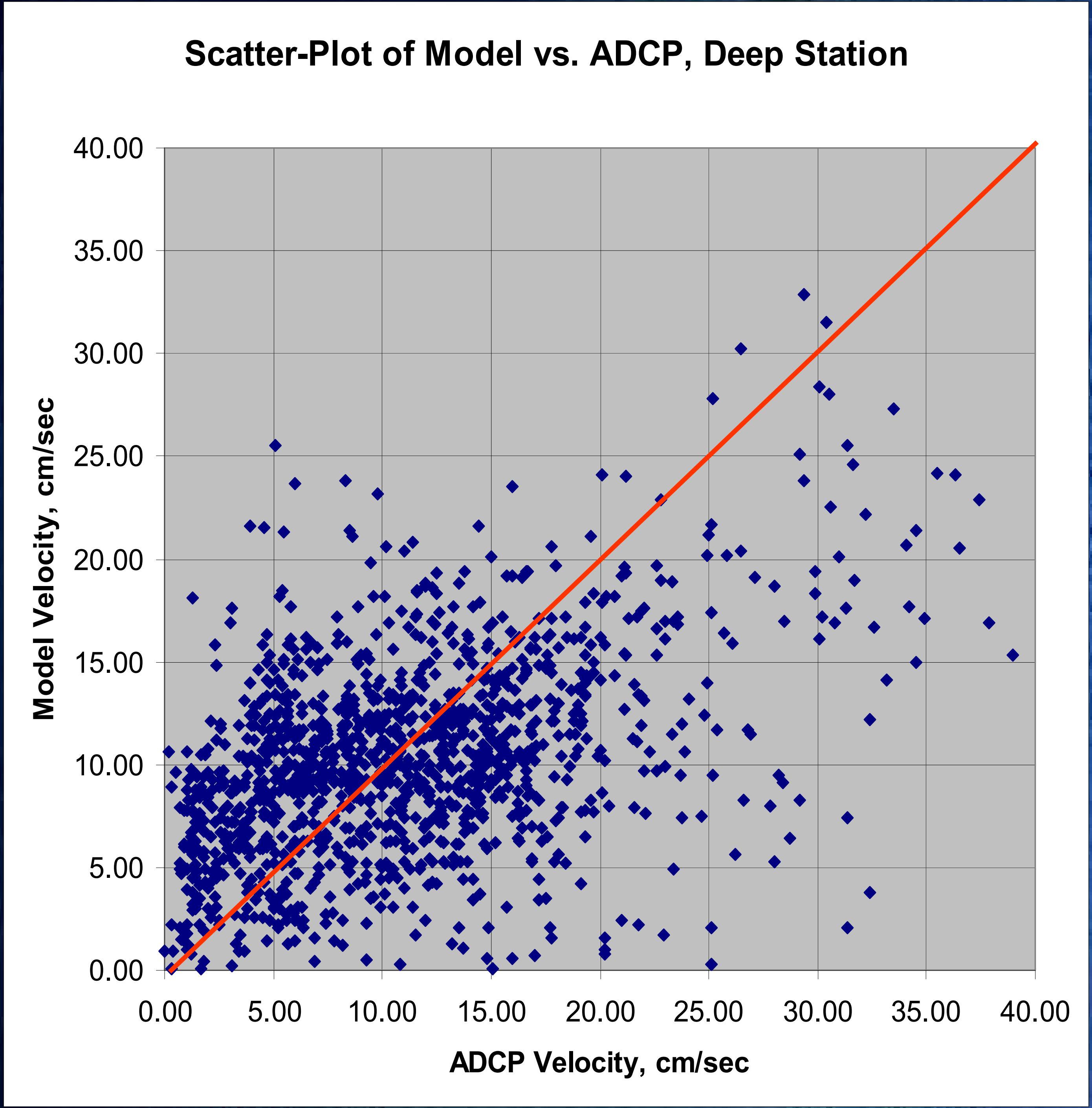


# Model Simulated Water Level Variations

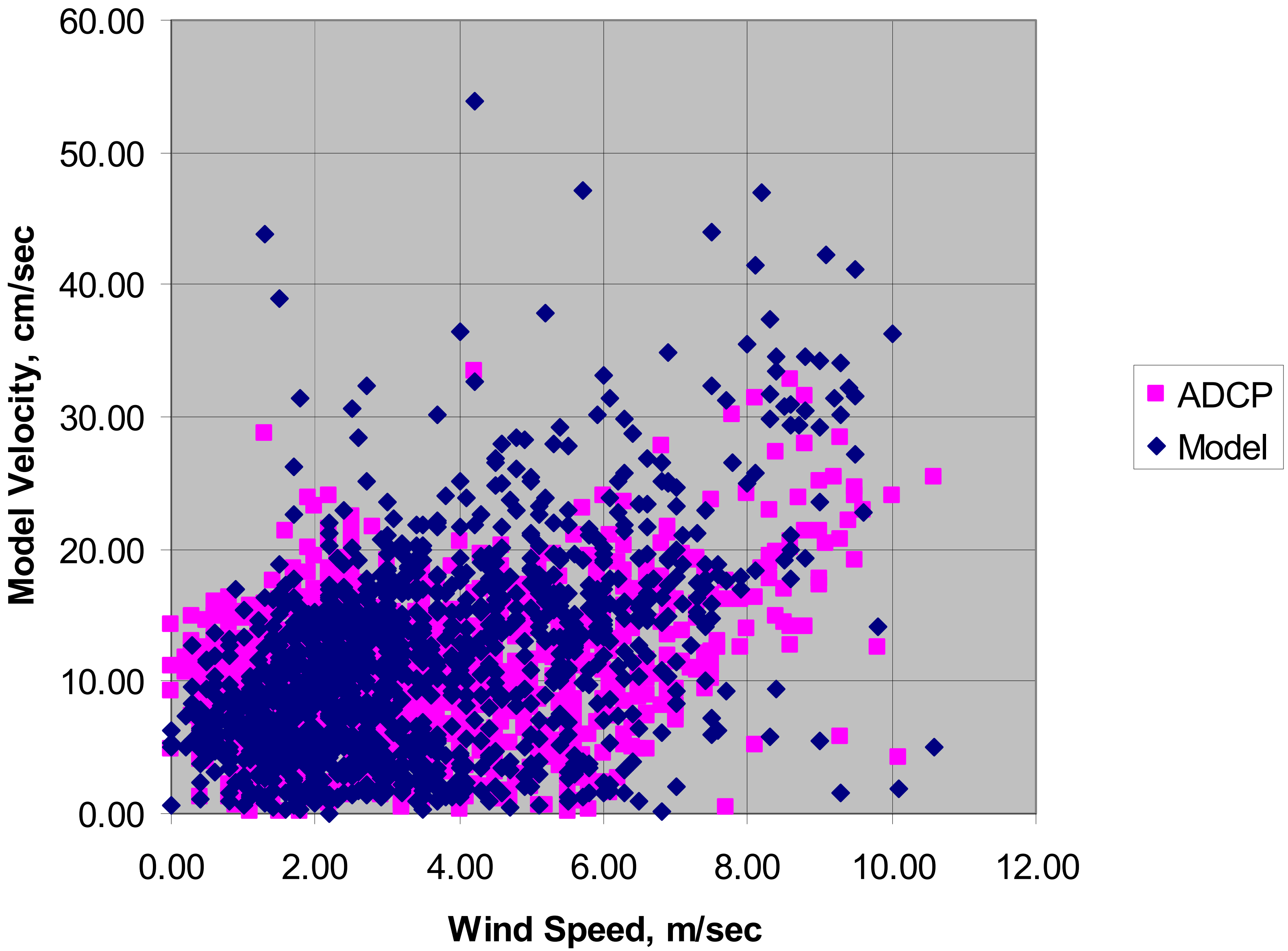


# Model Results vs. ADCP Observations at Deep (West) Station

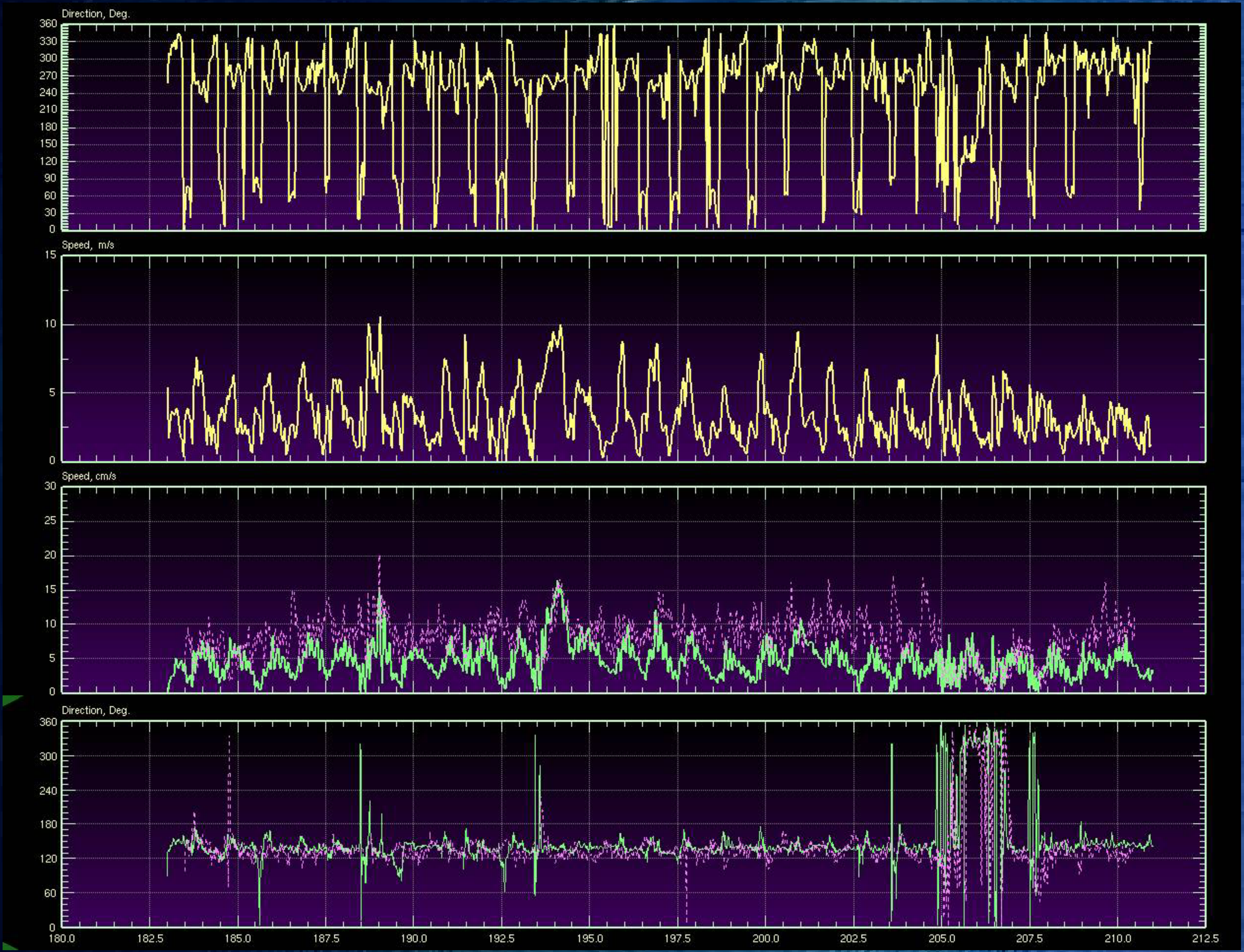


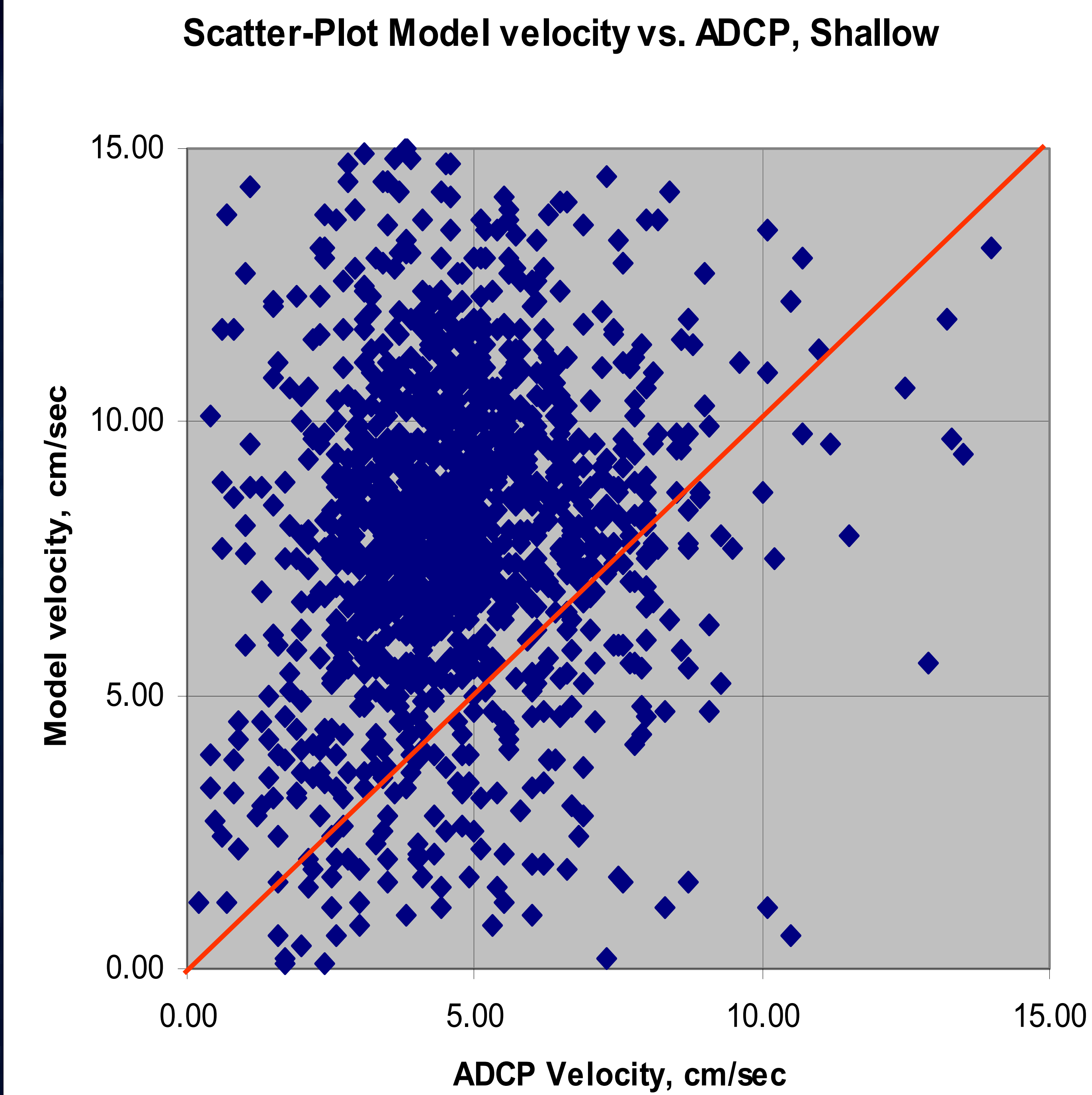

Scatter-Plot of Model vs. ADCP, Deep Station
Model Velocity, cm/sec
ADCP Velocity, cm/sec
0.00
5.00
10.00
15.00
20.00
25.00
30.00
35.00
40.00

# Wind Speed vs Velocities, Deep Station

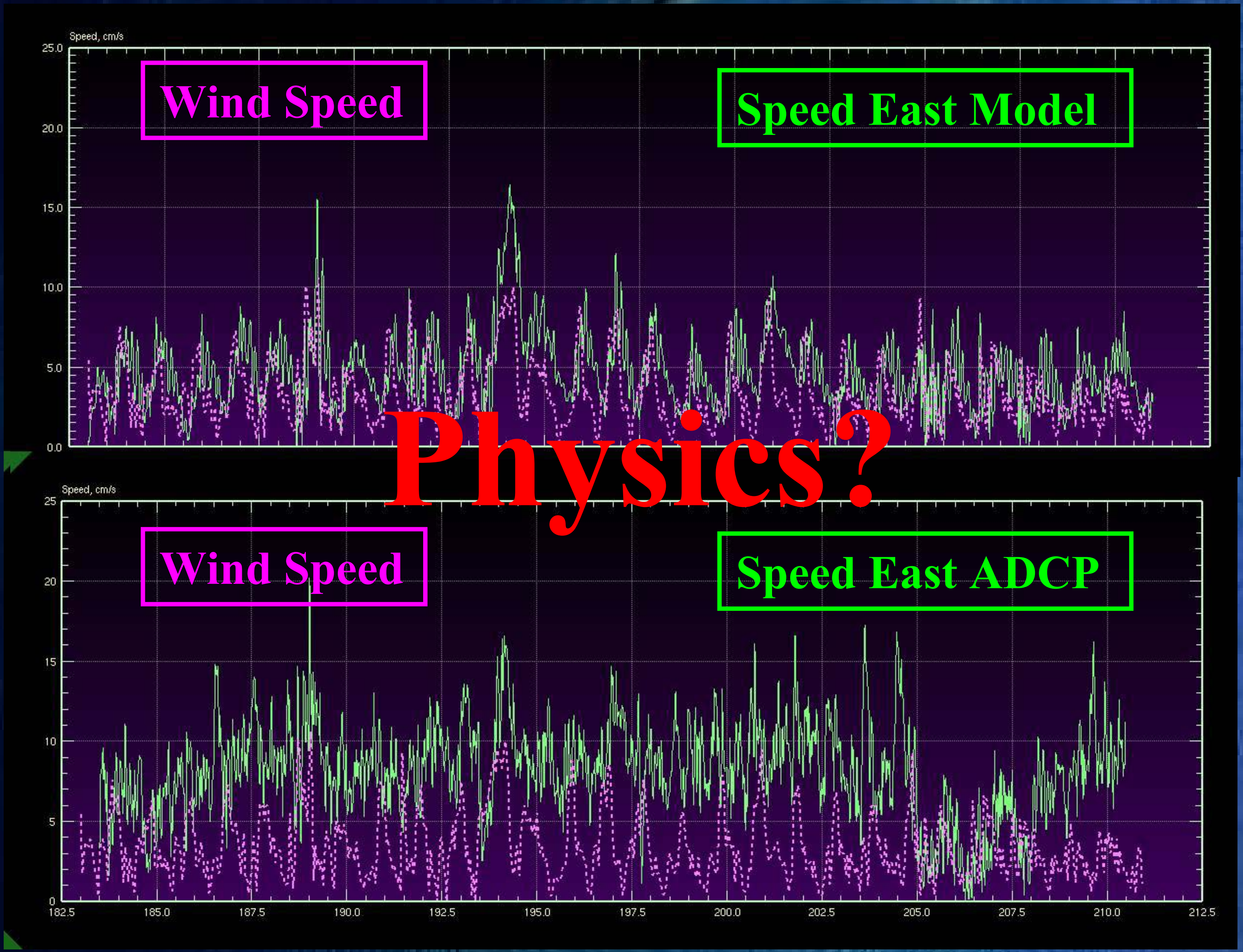


# Model Results vs. ADCP Observations at Shallow (East) Station

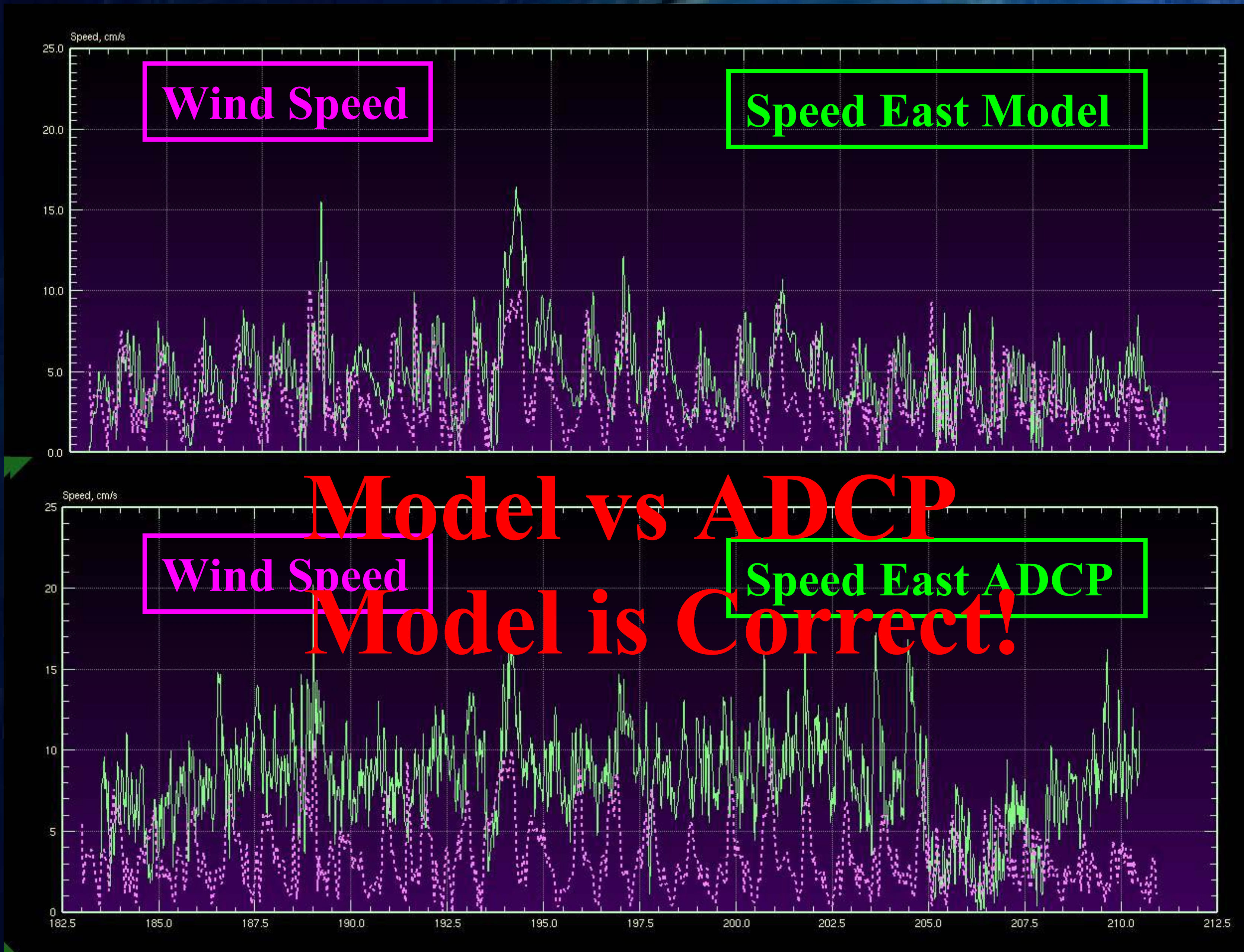



Scatter-Plot Model velocity vs. ADCP, Shallow
15.00
10.00
5.00
0.00
0.00
5.00
10.00
15.00
Model velocity, cm/sec
ADCP Velocity, cm/sec

# Correlations with wind speed



# Correlations with wind speed



## Take Home Message:

**Field Data Do not Necessarily Represent the Truth.**

**Field Data Must be Consistent with the Correct Physics!**

**There might be hidden messages in the data!**

# Conclusion

- The UnTRIM numerical model is used to reproduce the wind circulation in Upper Klamath Lake (UKL).
- Circulation in Upper Klamath Lake is shown to be completely controlled by wind.
- The ADCP data at a deep station is reproduced reasonably well; at the shallow station, data are shown to be suspect.
- Discrepancies are due to the inherent uncertainty in wind records which are used to drive the model

# Summary:

**Lagrangian VP shows clear Physics but difficult to Manage!**

**Eulerian VP is well suited for quantification!**

**Recommendation:**

**Think as a Lagrangian!**

**Act as an Eulerian!**

**Thank you!**